**Topic 4 Worksheet**

$D\_{4}$ is the symmetry group of the 4-gon, or the square.

Since $D\_{n}$ = $\left\{e,r,r^{2},…,r^{n-1}, f,rf,r^{2}f,…,r^{n-1}f\right\}$ with $θ$ = $\frac{360°}{n}$**,** $D\_{4}$ = $\left\{e, r, r^{2}, r^{3}, f, rf, r^{2}f , r^{3}f  \right\}$ with

 $θ$ = $\frac{360°}{4}$ = 90$°$.

**Problem 1:** Label the corners of the square undergoing the given symmetry transformations of $D\_{4}.$

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| --- | --- |
|  | The symmetry transformations of $D\_{4}$ are:e: no transformationr: clockwise rotation of 90$°$$r^{2}$: clockwise rotation of 180$°$ $r^{3}$: clockwise rotation of 270$°$ f: reflection about symmetry axisrf: clockwise rotation of 90$°$ and then reflection about the  symmetry axis$r^{2}f$: clockwise rotation of 180$°$  and then reflection about the symmetry axis$r^{3}f$: clockwise rotation of 270$°$  and then reflection about the symmetry axis |

**Problem 2:**$ $ Complete the Cayley table for $D\_{4}$. Use the Cayley table for $D\_{4}$ and the given transformation to show that $D\_{4}$ is equivalent to $C\_{8}.\*$

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| $D\_{4}$**Table** $D\_{4}$ = $\left\{e, r, r^{2}, r^{3}, f, rf, r^{2}f , r^{3}f  \right\}$

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **\*** | **e** | **r** | $$r^{2}$$ | $$r^{3}$$ | **f** | **rf** | $$r^{2}f$$ | $$r^{3}f$$ |
|  e |  |  |  |  |  |  |  |  |
| r |  |  |  |  |  |  |  |  |
| $$r^{2}$$ |  |  |  |  |  |  |  |  |
| $$ r^{3}$$ |  |  |  |  |  |  |  |  |
|  f |  |  |  |  |  |  |  |  |
| rf |  |  |  |  |  |  |  |  |
| $$r^{2}f$$ |  |  |  |  |  |  |  |  |
| $$r^{3}f$$ |  |  |  |  |  |  |  |  |

 | e $\rightarrow $er $\rightarrow $a$r^{2}$ $\rightarrow $b$r^{3}$ $\rightarrow $c$f\rightarrow $ drf$ \rightarrow f$$r^{2}f$ $\rightarrow g$$r^{3}f$ $\rightarrow h$ | $C\_{8}$**Table** $C\_{8}=\left\{e,a,b,c,d,f,g,h\right\}$

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **\*** | e | a | b | c | d | f | g | h |
| e |  |  |  |  |  |  |  |  |
| a |  |  |  |  |  |  |  |  |
| b |  |  |  |  |  |  |  |  |
| c |  |  |  |  |  |  |  |  |
| d |  |  |  |  |  |  |  |  |
| f |  |  |  |  |  |  |  |  |
| g |  |  |  |  |  |  |  |  |
| h |  |  |  |  |  |  |  |  |

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Is $D\_{4}$ abelian?

\* See Topic 3 Homework Problems for $C\_{8}.$