**Topic 5 Worksheet**

Since the number of permutations of n objects is equal to n!, the number of permutations of the four numbers 1,2,3, and 4 is equal to 4! = 4 3 2 1 = 24.

The symmetric group is composed of***eight of these twenty - four***permutations. The eight permutations in are: , , and .

**Problem 1:** Construct the eight permutation matrices of as defined by the given mappings.

e: a: b: c:

d: f: g: h:

**Problem 2:** Use the permutation matrices of and the operation of composition to complete the Cayley table for .

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| **Table**   |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | |  | e | a | b | c | d | f | g | h | | e |  |  |  |  |  |  |  |  | | a |  |  |  |  |  |  |  |  | | b | b | c | e | a | g | h | d | f | | c | c | e | a | b | h | d | f | g | | d | d | h | g | f | e | c | b | a | | f | f | d | h | g | a | e | c | b | | g | g | f | d | h | b | a | e | c | | h | h | g | f | d | c | b | a | e | |

**Problem 3:** is the symmetry group of the square. is equivalent to . Write the eight symmetries of the square as matrices.

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**Problem 4:** Use the Cayley table for and the given transformation to show that is equivalent to

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| **Table**  =   |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | | **\*** | **e** | **r** |  |  | **f** | **rf** |  |  | | e | e | r |  |  | f | rf | f | f | | r | r |  |  | e | rf | f | f | f | |  |  |  | e | r | f | f | f | rf | |  |  | e | r |  | f | f | rf | f | | f | f | f | f | rf | e |  |  | r | | rf | rf | f | f | f | r | e |  |  | |  | f | rf | f | f |  | r | e |  | |  | f | f | rf | f |  |  | r | e | | e e  r    c  f d  f  g  h | **Table**   |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | | **\*** | e | a | b | c | d | f | g | h | | e |  |  |  |  |  |  |  |  | | a |  |  |  |  |  |  |  |  | | b |  |  |  |  |  |  |  |  | | c |  |  |  |  |  |  |  |  | | d |  |  |  |  |  |  |  |  | | f |  |  |  |  |  |  |  |  | | g |  |  |  |  |  |  |  |  | | h |  |  |  |  |  |  |  |  | |