**Topic 5 Worksheet**

Since the number of permutations of n objects is equal to n!, the number of permutations of the four numbers 1,2,3, and 4 is equal to 4! = 4 $×$ 3 $×$ 2 $× $1 = 24.

The symmetric group $S\_{4}=\left\{1,2,3,4\right\}$ is composed of***eight of these twenty - four***permutations. The eight permutations in $S\_{4}$ are: $\left\{1,2,3,4\right\}$, $\left\{4,1,2,3\right\}, \left\{3,4,1,2\right\}, \left\{2,3,4,1\right\}, \left\{2,1,4,3\right\}, \left\{1,4,3,2\right\},\left\{4,3,2,1\right\}$, and $\left\{3,2,1,4\right\}$.

**Problem 1:** Construct the eight permutation matrices of $S\_{4} $as defined by the given mappings.

e: $\left\{1,2,3,4\right\}$ $⟶$ $\left\{1,2,3,4\right\}$ a: $\left\{1,2,3,4\right\}$ $⟶$ $\left\{4,1,2,3\right\}$ b: $\left\{1,2,3,4\right\}$ $⟶$ $\left\{3,4,1,2\right\} $ c: $\left\{1,2,3,4\right\}$ $⟶$ $\left\{2,3,4,1\right\}$

d: $\left\{1,2,3,4\right\}$ $⟶$ $\left\{2,1,4,3\right\}$ f: $\left\{1,2,3,4\right\}$ $⟶$ $\left\{1,4,3,2\right\}$ g: $\left\{1,2,3,4\right\}$ $⟶$ $\left\{4,3,2,1\right\} $ h: $\left\{1,2,3,4\right\}$ $⟶$ $\left\{3,2,1,4\right\}$

**Problem 2:**$ $ Use the permutation matrices of $S\_{4}$ and the operation of composition to complete the Cayley table for $S\_{4}$.

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| $S\_{4}$**Table**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $$∘$$ | e | a | b | c | d | f | g | h |
|  e |  |  |  |  |  |  |  |  |
| a |  |  |  |  |  |  |  |  |
| b | b  | c | e | a | g | h | d | f |
| c | c | e | a | b | h | d | f | g |
| d | d | h | g | f | e | c | b | a |
| f | f | d | h | g | a | e | c | b |
| g | g | f | d | h | b | a | e | c |
| h | h | g | f | d | c | b | a | e |

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**Problem 3:** $S\_{4}$ is the symmetry group of the square. $S\_{4}$ is equivalent to $D\_{4}$. Write the eight symmetries of the square as matrices.

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**Problem 4:**$ $ Use the Cayley table for $D\_{4}$ and the given transformation to show that $S\_{4}$ is equivalent to $D\_{4}.$

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| $D\_{4}$**Table** $D\_{4}$ = $\left\{e, r, r^{2}, r^{3}, f, rf, r^{2}f , r^{3}f  \right\}$

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **\*** | **e** | **r** | $$r^{2}$$ | $$r^{3}$$ | **f** | **rf** | $$r^{2}f$$ | $$r^{3}f$$ |
|  e | e | r | $$r^{2}$$ | $$r^{3}$$ | f | rf | $r^{2}$f | $r^{3}$f |
| r | r | $$r^{2}$$ | $$r^{3}$$ | e | rf | $r^{2}$f | $r^{3}$f | f |
| $$r^{2}$$ | $$r^{2}$$ | $$r^{3}$$ | e | r | $r^{2}$f | $r^{3}$f | f | rf |
| $$ r^{3}$$ | $$r^{3}$$ | e | r | $$r^{2}$$ | $r^{3}$f | f | rf | $r^{2}$f |
|  f | f | $r^{3}$f | $r^{2}$f | rf | e | $$r^{3}$$ | $$r^{2}$$ | r |
| rf | rf | f | $r^{3}$f | $r^{2}$f | r | e | $$r^{3}$$ | $$r^{2}$$ |
| $$r^{2}f$$ | $r^{2}$f | rf | f | $r^{3}$f | $$r^{2}$$ | r | e | $$r^{3}$$ |
| $$r^{3}f$$ | $r^{3}$f | $r^{2}$f | rf | f | $$r^{3}$$ | $$r^{2}$$ | r | e |

 | e$ \rightarrow $ er $\rightarrow a$$r^{2}\rightarrow b$ $r^{3}$ $\rightarrow $ cf $\rightarrow $ d$rf$ $\rightarrow $ f$r^{2}f$ $\rightarrow $ g$r^{3}f$ $\rightarrow $ h | $S\_{4}$**Table**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **\*** | e | a | b | c | d | f | g | h |
| e |  |  |  |  |  |  |  |  |
| a |  |  |  |  |  |  |  |  |
| b |  |  |  |  |  |  |  |  |
| c |  |  |  |  |  |  |  |  |
| d |  |  |  |  |  |  |  |  |
| f |  |  |  |  |  |  |  |  |
| g |  |  |  |  |  |  |  |  |
| h |  |  |  |  |  |  |  |  |

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