COMP 9121 Assignment 1 (2020)

Due: TBD

Question 1 (Two-dimension Parity Check, 20%)**.** Consider the following two-dimension parity check, with original data in a 4×4 matrix. For each row and column, we generate a parity bit, forming a 5×5 matrix. The parity bits are in the last column and last row. Then, the 25 bits will be transmitted through a bit-flipping channel. For simplification, we assume that each original bit is flipped with probability p independently; p is a small probability; **the parity bits are not flipped at all.**

(1) [3%] Assume the following bits are sent:

$$\begin{matrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix} \qquad (1)$$

and the following bits are received

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix} \qquad (1)$$

Then, what does the receiver do and why?

(2) [3%] The example in (1) shows one case that the receiver believes there is one bit error and the error can be corrected, but the fixed bits are still wrong. Please give another example: (a) there are 9 bits flipped; (b) the receiver believes there is one bit error and the error can be corrected; and (c) the fixed bits are still wrong.

(3) [3%] If two original data bits are flipped, can the receive detect errors? can the receiver recover the errors? Why or why not.

(4) [3%] Give an example where there are bit error(s) but the receiver cannot even detect the error(s).

(5) [8%+ up to 4% bonus] Calculate the following probabilities (as a function of p): (a) The receiver believes there is one bit error and the error can be corrected, and the fixed bits are correct. (b) The receiver believes there is one bit error and the error can be corrected, but the fixed bits are wrong. For (b), you will get full mark if your answer is sufficiently close to the accurate solution (i.e., consider those “common” error patterns but ignore those “rare” error patterns). Bonus marks will be given if your answer is more accurate.

Question 2 (CRC, 15%)**.** In this question, you are going to test the performance of CRC code. Let $N = 8$, 9, and 10, and $G = 1001$. The length of coded bits is $N + 3$. To figure out the problem, you can choose one of the following two approaches (A1 or A2). You are encouraged to work on both approaches to verify your results. Your aim is to fill the following tables.

TABLE I
$N = 8$

p	0	0.02	0.04	0.06	0.08	0.1
p_A						
p_B						
p_C						

TABLE II
$N = 9$

p	0	0.02	0.04	0.06	0.08	0.1
p_A						
p_B						
p_C						

TABLE III
$N = 10$

p	0	0.02	0.04	0.06	0.08	0.1
p_A						
p_B						
p_C						

(A1) Implement a simulator of cyclic redundancy check (CRC) code and test its performance. You need to submit your Python code if you choose do so. You can reuse parts of codes in Lab. The procedure is summarized as follows:

(1) Randomly generate N-bit data. You need to set $N = 8$, 9, and 10. (You can reuse the code in Lab.)

(2) Using the generator $G = 1001$, generate the CRC bits and derive an $(N+3)$-bit coded result. (Hint: implement long division in this step.)

(3) Send the coded bits into a random flipping channel with bit-flip probability p. You need to test different values of p.

(4) The receiver checks the received bits. (Hint: implement long division in this step). Then, there will be three possibilities:

- Event A: None of the bits are flipped.
- Event B: Some of the bits are flipped, and this is detected by the CRC.
- Event C: Some of the bits are flipped, but this is not detected by the CRC.

(5) Repeat the above procedure many times (e.g., 100000 or more). Find out the probabilities of A), B), and C), i,e., p_A, p_B, and p_C

(A2) Theoretically analyse the performance of CRC code. You need to provide your detailed analysis if you choose to do so. Compute the theoretical probabilities of A, B, and C. Use the theoretical results to fill the table. (Hint: Please consider the error patterns divisible by 1001. Approximations can be made in this step, i.e., consider those “common” error patterns but ignore those “rare” error patterns.)

Question 3 (CSMA Performance, 15%). $M+N$ computers have been connected in a network as illustrated. M computers are in the left side and N computers are in the right side. CSMA-CD is used. X_0 is a switch. X_1 and X_2 are hubs. The length of each link is written in meters. Each computer generates 500 packets per second with each packet being 1000 bytes. The maximum rate of all links is 1 Gbps. The propagation speed in the medium is 2.0×10^8 meters/second.

For the M computers, $\frac{2}{3}$ traffic is kept locally, $\frac{1}{3}$ traffic goes to the other side. For the N computers, $\frac{1}{3}$ traffic is kept locally, $\frac{2}{3}$ traffic goes to the other side.

What is the max number of the computers that can be supported in the network, i.e., what is the max value of $M+N$?