**Principles of Investments**

**Learning Exercise 2**

**Bond Concepts**

**N. Gershun**

**Part 1: Theoretical Problems**

1. Suppose that the price of discount (zero coupon) bonds maturing in years 1, 2, 3, 4, and 5 are given (respectively) by

|  |  |
| --- | --- |
| Price | Time to Maturity |
| 940 | **1** |
| 870 | **2** |
| 800 | **3** |
| 715 | **4** |
| 630 | **5** |

Consider the following risk-free investments. Which is best?

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | T=0 | 1 | 2 | 3 | 4 | 5 |
| Investment A | -40 | 20 | 15 | 10 | 5 | 1 |
| Investment B | -10 | 1 | 5 | 10 | 15 | 20 |

2. Three zero coupon risk-free discount bonds of one, two and three year term to maturity are selling for, respectively, $950, $890 and $800. What would be the selling price today of a 10% coupon bond of 3 year maturity (maturity value $1,000)?

3. Consider a coupon bond, with the current period t = 0 market price $900, with payments:

|  |  |  |  |
| --- | --- | --- | --- |
| $$t=0$$ | **1** | **2** | **3** |
|  | 50 | 50 | 1050 |

 Discount (zero coupon) bonds of 1, 2 and 3 years maturity (all with maturity value of $1000) sell for respectively, 960, 900, 820 dollars. Is this coupon bond properly priced? If not, design an arbitrage argument to profit by the mispricing.

1. The prices of discount bonds (all with maturity value of $1,000) maturing in years 1, 2, 3, 4, 5 are given below.

|  |  |
| --- | --- |
| Price | Time to Maturity |
| 920 | **1** |
| 860 | **2** |
| 790 | **3** |
| 700 | **4** |
| 600 | **5** |

What is the yield to maturity on a risk-free 5% bond due in 5 years (also with maturity value of $1,000)?

**Part 2 – Empirical Investigation of the Term Structure of Interest Rates**

Prior to working on this part of LE 2, you need to:

1. Study lectures 2A and 2B
2. Study a Review file
3. Watch a video with the Excel example of Bootstrapping
4. Explore the Excel file used in this video

**Step 1: Data**

Download data on prices, times to maturity and coupon rates for SIX government bonds maturing in consecutive six-month intervals. Make sure that the accrued interest on the bonds, which you select, is as small as possible. Recall that Treasury bonds pay coupons either on the 15th of the month or on the last day of the month every six months. For example, if today’s date is June 5, 2020 and you decide to download data for your bonds today, the first bond on your list should mature on November 30, 2020, the second bond --on May 31, 2021, the third bond on November 30, 2021, etc. These bonds paid their coupons on May 31st, i.e. just five days ago. The accrued interest on these bonds is negligible and you can ignore it in your calculations. If you choose bonds that mature either on June 15th of any year or December 15th of any year, these bonds will pay their next coupon on June 15, 2020, i.e. in ten days. The accrued interest on these bonds is very large and cannot be ignored.

The best source of data for this project is Bloomberg. If you are unable to use Bloomberg, use Wall Street Journal data:

<https://www.wsj.com/market-data/bonds/treasuries>

**Step 2: Calculations**

Using the bootstrapping method, calculate the first six elements of the term structure of interest rates. Remember, that US Treasury bonds pay semi-annual coupons and you need to adjust your calculations to take semi-annual coupons into account. The technique is exactly the same as with annual coupons, but your time period is six months. You will find semi-annual rates. Annualize the rates. In your report for this part of the LE. You essentially need to replicate the example in the video and the corresponding Excel file, posted on Bb.

**Step 3: Deliverables**:

For all deliverables you need to show formulas in symbols (not Excel formulas) as is done in lectures. Then show how you plug your data into these formulas. Write short verbal comments at the end of each step.

1. Show how you convert annual coupon rates into semi-annual coupon payments in dollars.
2. Show how to convert the ask price (you need to use the ask price for this project) from ticks into the dollar price.
3. Create a time line (it will be a triangular table with each row showing cash flows of one bond).
4. Show all your calculations for each of the six rates; include formulas and show how you plugged in your data into these formulas. You can use Equation Editor in Word to type any formula.
5. Attach printouts with your data from Bloomberg or WSJ to your report as an Appendix.
6. Using Excel Chart menu, draw the graph of the term structure, with time periods on the horizontal axis and term structure rates on the vertical axis. Is the term structure currently increasing or decreasing? Discuss briefly.
7. Briefly comment on the information that the observed term structure implies for economic conditions, as perceived by market participants.

Show all your calculations, including all regular (not Excel) formulas, graph and explanations. Report (including all the equations) should be typed.