1. **(2 pts)** True or False? The function $f(x) = x^2 \sin \frac{1}{x}$ has an removable discontinuity at $x = 0$.

2. **(2 pts)** True or False? It is possible that a function $f(x)$ with domain $\mathbb{R}$ is continuous at only one point.

3. **(5 pts)** Suppose that $g(x)$ is a function which is defined for every x near 0, except possibly at $x = 0$. Furthermore, suppose that for every such x, the inequality

$$\frac{\sin x}{x} \leq g(x) \leq 2 - \cos x$$

is satisfied. What can you say about the limit

$$\lim_{x \to 0} g(x)?$$

Choose the best answer.

(a) $\lim_{x\to 0} g(x) = 0$ by the Squeeze Theorem.
(b) $\lim_{x\to 0} g(x) = 1$ by the Squeeze Theorem.
(c) $\lim_{x\to 0} g(x) = 2$ by the Squeeze Theorem.
(d) That the limit does not exist.
(e) Nothing from the given information. The limit may or may not exist.

4. **(6 pts)** What does the Intermediate Value Theorem (IVT) tell us about the following function?

$$f(x) = 3x^4 - 5x^2 + 1$$

Choose the best answer. Show any work in obtaining your answer.

(a) $f(x)$ has a zero in $(0, 1)$.
(b) $f(x)$ has a zero in $(1, 2)$.
(c) $f(x)$ has a zero in each of the intervals $(0, 1)$ and $(1, 2)$.
(d) None of the above.

5. **(6 pts)** Evaluate the limit in a step by step fashion. Justify each step by indicating the appropriate Limit Law.

$$\lim_{x \to 3} \sqrt{2x^2 - \frac{6}{x}}$$

6. **(6 pts)** Find the numbers at which f is discontinuous. At which of these numbers is f continuous from the left, from the right, or neither?

$$f(x) = \begin{cases} x+2 & x \leq 0 \\ x^2+1 & 0 < x \leq 1 \\ \frac{2}{x} & 1 < x \end{cases}$$

7. **(18 pts)** Evaluate the following limits:

(a)

$$\lim_{t \to 3} \frac{\sqrt{t^2+16}-5}{t-3}$$

(b)

$$\lim_{x \to 0} \frac{\frac{1}{1-2x} - \frac{1}{1+2x}}{2x}$$

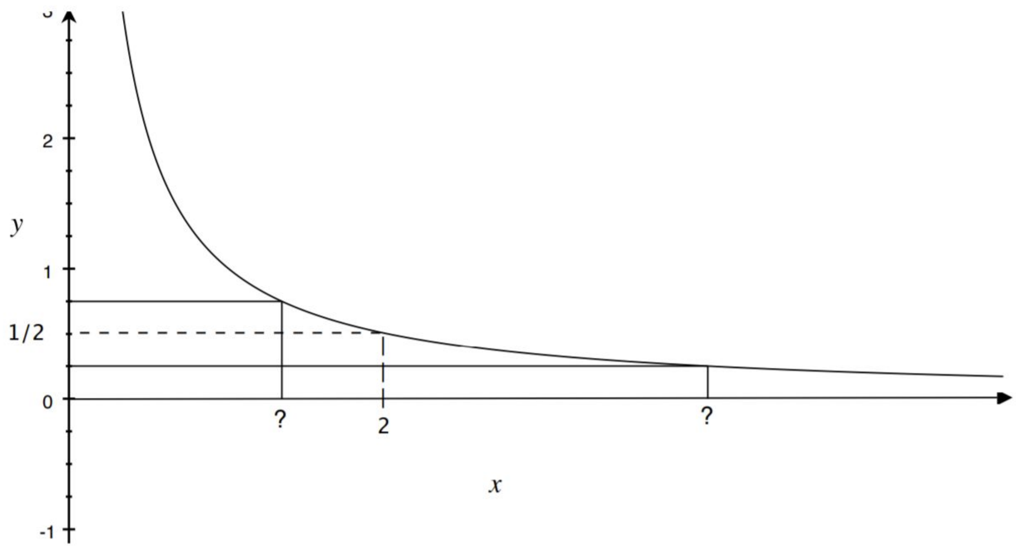
(c)

$$\lim_{x \to 0^+} \frac{1}{x^2 - x}$$

9. **(6 pts)**

Use the given graph of $y = \frac{1}{x}$ to find a number δ such that

$$\text{if } 0 < |x - 2| < \delta, \text{ then } |\tfrac{1}{x} - \tfrac{1}{2}| < \tfrac{1}{4}.$$



8. **(9 pts)**

(a) Let

$$f(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$$

Determine the following limits:

(i) $\lim_{x \to 0+} f(x)$

(ii) $\lim_{x \to 0-} f(x)$

(iii) $\lim_{x \to 0} f(x)$

(b) Let $f(x)$ be the same function as in part (a), and let

$$g(x) = f(\sin x)$$

First, find the following limits:

(i) $\lim_{x \to \pi+} g(x)$

(ii) $\lim_{x \to \pi-} g(x)$

Then state whether g is continuous at $x = \pi$, and explain based on the definition of continuity at a point.