Midterm Exam 1

In this Exam, you perform the primal simplex method. Use Dantzig’s rule to choose entering columns through the exam. Instead of floating point arithmetic, it is recommended to do arithmetic with fractions. Solve the problems using the methods you learned in class (Optional ways you google will not earn any point). Do not submit Excel Worksheets except Problem 6.

NAME:

SIGN:

Total: /25

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| **1-1** | **1-2** | **2-1** | **2-2** | **3-1** |
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| **3-2** | **3-3** | **4** | **5** | **6** |
| **/2** | **/2** | **/3** | **/3** | **/5** |

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1. Perform the primal simplex method to solve the following LPs. The slack variables are the initial basic variables. Write the basis and the simplex tableau at each iteration. Fill in the blanks of the tableaux and the under-lined blanks.

1-1. (2 points)

 Maximize 4 x1 + 6 x2,

 Subject to – x1 + x2 ≤ 11,

 x1 + x2 ≤ 27,

 2 x1 + 5 x2 ≤ 90,

 x1 , x2 ≥ 0.

The Simplex Tableau with respect to B0 = {s1, s2, s3}:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | s1 | s2 | s3 | Max |   |
| 0th row | 4 | 6 | 0 | 0 | 0 |  Z - 0 | ratio |
| 1st row | -1 | 1 | 1 | 0 | 0 | 11 | 11 |
| 2nd row | 1 | 1 | 0 | 1 | 0 | 27 | 27 |
| 3rd row | 2 | 5 | 0 | 0 | 1 | 90 | 18 |

What is the objective value? What is the basic feasible solution? Is the basis optimal?

The simplex Tableau with respect to B1 = {x2, s2, s3}:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | s1 | s2 | s3 | Max |   |
| 0th row | 10 | 0 | -6 | 0 | 0 | Z – 66  | ratio |
| 1st row | -1 | 1 | 1 | 0 | 0 | 11 |  |
| 2nd row | 2 | 0 | -1 | 1 | 0 | 16 |  |
| 3rd row | 7 | 0 | -5 | 0 | 1 | 35 |  |

What is the objective value? What is the basic feasible solution? Is the basis optimal?

* 1. (Continued)

The simplex Tableau with respect to B2 = { }:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | s1 | s2 | s3 | Max |   |
| 0th row |  |  |  |  |  |  | ratio |
| 1st row |  |  |  |  |  |  |  |
| 2nd row |  |  |  |  |  |  |  |
| 3rd row |  |  |  |  |  |  |  |

What is the objective value? What is the basic feasible solution? Is the basis optimal?

The simplex Tableau with respect to B3 = { }:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | s1 | s2 | s3 | Max |   |
| 0th row |  |  |  |  |  |  | ratio |
| 1st row |  |  |  |  |  |  |   |
| 2nd row |  |  |  |  |  |  |   |
| 3rd row |  |  |  |  |  |  |   |

What is the objective value? What is the basic feasible solution? Is the basis optimal?

1-2. (2 points)

 Maximize 2 x1 – x2 + x3,

 Subject to 3 x1 + x2  + x3 ≤ 60,

 x1 – x2 + 2 x3 ≤ 10,

 x1 + x2 – x3 ≤ 20,

 x1 , x2 , x3 ≥ 0.

The Simplex Tableau with respect to B0 = {s1, s2, s3}:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | x3 | s1 | s2 | s3 | Max |   |
| 0th row | 2 | -1 | 1 | 0 | 0 | 0 | Z – 0  | ratio |
| 1st row | 3 | 1 | 1 | 1 | 0 | 0 | 60 |  |
| 2nd row | 1 | -1 | 2 | 0 | 1 | 0 | 10 |  |
| 3rd row | 1 | 1 | -1 | 0 | 0 | 1 | 20 |  |

What is the objective value? What is the basic feasible solution? Is the basis optimal?

1-2 (Contnued) The Simplex Tableau with respect to B1 = { }:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | x3 | s1 | s2 | s3 | Max |   |
| 0th row |  |  |  |  |  |  |  | ratio |
| 1st row |  |  |  |  |  |  |  |  |
| 2nd row |  |  |  |  |  |  |  |  |
| 3rd row |  |  |  |  |  |  |  |  |

What is the objective value? What is the basic feasible solution? Is the basis optimal?

The simplex Tableau with respect to B2 = { }:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | x3 | s1 | s2 | s3 | Max |   |
| 0th row |  |  |  |  |  |  |  | ratio |
| 1st row |  |  |  |  |  |  |  |   |
| 2nd row |  |  |  |  |  |  |  |   |
| 3rd row |  |  |  |  |  |  |  |   |

What is the objective value? What is the basic feasible solution? Is the basis optimal?

2. Fill in the blanks of the tableaux and the under-lined blanks.

2-1. (2 points) Compute the simplex tableau corresponding to **initial basis = { x1, x4, x3}** and then perform the simplex iteration; i.e., the column of the most negative reduced cost enters into basis.

 min x1 + 3 x2 + 3 x3 +2 x4 – 2 x5,

s t – 4 x2 + 3 x3 – x4 = 1,

 4 x1 + 3 x2 + x4 + x5 = 2,

 – 3 x1 + 2 x2 + x4 + x5 = 2,

  x1, x2, x3, x4, x5 ≥ 0.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | x3 | x4 | x5 | Min |  |
| 0th row | 1 | 3 | 3 | 2 | -2 | 0  | Ratio |
| 1st row | 0 | -4 | 3 | -1 | 0 | 1 |  |
| 2nd row | 4 | 3 | 0 | 1 | 1 | 2 |  |
| 3rd row | -3 | 2 | 0 | 1 | 1 | 2 |  |

2.1 (Continued)

The Simplex Tableau with respect to **initial basis B0 = { x1, x4, x3}**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | x3 | x4 | x5 | Min |  |
| 0th row |  |  |  |  |  |  | ratio |
| 1st row |  |  |  |  |  |  |  |
| 2nd row |  |  |  |  |  |  |  |
| 3rd row |  |  |  |  |  |  |  |

What is the objective value? What is the basic feasible solution? Is the basis optimal?

The Simplex Tableau with respect to **B1 = { }**.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | x3 | x4 | x5 | min |  |
| 0th row |  |  |  |  |  |  | ratio |
| 1st row |  |  |  |  |  |  |   |
| 2nd row |  |  |  |  |  |  |   |
| 3rd row |  |  |  |  |  |  |   |

What is the objective value? What is the basic feasible solution? Is the basis optimal? .

2-2. (2 points) Compute the simplex tableau corresponding to **initial basis = { x1, x2, x3}** and then perform the simplex iteration; i.e., the column of the most negative reduced cost enters into basis.

 min 3 x1 + 2 x2 + 5 x3 + 8 x4,

s t 2 x2 + 3 x3 – x5 = 6,

 4 x1 + 2 x2 + 2 x3 + 4 x4 – x6 = 10,

 x1 + 4 x2 + 2 x3 + 5 x4 – x7 = 8,

 x1, x2, x3, x4, x5, x6, x7 ≥ 0.

Coefficient Matrix

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | x3 | x4 | x5 | x6 | x7 | min |   |
| 0th row |  |  |  |  |  |  |  |  | ratio |
| 1st row |  |  |  |  |  |  |  |  |  |
| 2nd row |  |  |  |  |  |  |  |  |  |
| 3rd row |  |  |  |  |  |  |  |  |  |

2-2 (Continued)

The Simplex Tableau with respect to **initial basis B0 = { x1, x2, x3}**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | x3 | x4 | x5 | x6 | x7 | min |   |
| 0th row |  |  |  |  |  |  |  |  | ratio |
| 1st row |  |  |  |  |  |  |  |  |  |
| 2nd row |  |  |  |  |  |  |  |  |  |
| 3rd row |  |  |  |  |  |  |  |  |  |

What is the objective value? What is the basic feasible solution? Is the basis optimal?

The Simplex Tableau with respect to **B1 = { }**.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | x3 | x4 | x5 | x6 | x7 | min |   |
| 0th row |  |  |  |  |  |  |  |  | ratio |
| 1st row |  |  |  |  |  |  |  |  |   |
| 2nd row |  |  |  |  |  |  |  |  |   |
| 3rd row |  |  |  |  |  |  |  |  |   |

What is the objective value? What is the basic feasible solution? Is the basis optimal?

3. Consider the following linear programming problem:

 min 10 x1 + 2 x2 + 8 x3,

 st 2 x1 – x2 + 2 x3 ≥ 4,

 x1 + x2 + x3 ≥ 6,

 x1 + 2 x3 ≥ 8,

 x1, x2, x3  ≥ 0

3-1. (2 points) Transform the LP into a standard form employing surplus variables s1, s2, and s3 ≥ 0.

3-2. (2 points) Formulate and solve Phase 1 employing artificial variables a1, a2, and a3 ≥ 0. What is the optimal basis?

The Coefficient Matrix:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x1 | x2 | x3 | s1 | s2 | s3 | a1 | a2 | a3 | min |   |
|  |  |  |  |  |  |  |  |  |  | ratio |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |

The Simplex Tableau with respect to **initial basis B0 = { a1, a2, a3}:**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x1 | x2 | x3 | s1 | s2 | s3 | a1 | a2 | a3 | min |   |
|  |  |  |  |  |  |  |  |  |  | ratio |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |

What is the objective value? What is the basic feasible solution? Is the basis optimal?

3-2 (Continued)

The Simplex Tableau with respect to **B1 = { }:**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x1 | x2 | x3 | s1 | s2 | s3 | a1 | a2 | a3 | min |   |
|  |  |  |  |  |  |  |  |  |  | ratio |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |

What is the objective value? What is the basic feasible solution? Is the basis optimal?

The Simplex Tableau with respect to **B2 = { }:**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x1 | x2 | x3 | s1 | s2 | s3 | a1 | a2 | a3 | min |   |
|  |  |  |  |  |  |  |  |  |  | ratio |
|  |  |  |  |  |  |  |  |  |  |   |
|  |  |  |  |  |  |  |  |  |  |   |
|  |  |  |  |  |  |  |  |  |  |   |

What is the objective value? What is the basic feasible solution? Is the basis optimal?

3-2 (Continued if needed)

The Simplex Tableau with respect to **B = { }:**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x1 | x2 | x3 | s1 | s2 | s3 | a1 | a2 | a3 | min |   |
|  |  |  |  |  |  |  |  |  |  | ratio |
|  |  |  |  |  |  |  |  |  |  |   |
|  |  |  |  |  |  |  |  |  |  |   |
|  |  |  |  |  |  |  |  |  |  |   |

What is the objective value? What is the basic feasible solution? Is the basis optimal?

The Simplex Tableau with respect to **B = { }:**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x1 | x2 | x3 | s1 | s2 | s3 | a1 | a2 | a3 | min |   |
|  |  |  |  |  |  |  |  |  |  | ratio |
|  |  |  |  |  |  |  |  |  |  |   |
|  |  |  |  |  |  |  |  |  |  |   |
|  |  |  |  |  |  |  |  |  |  |   |

What is the objective value? What is the basic feasible solution? Is the basis optimal?

3-3. (2 points) Solve Phase 2 beginning with the initial basis obtained from 3-2 above.

The Simplex Tableau with respect to **B0 = { }:**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x1 | x2 | x3 | s1 | s2 | s3 | min |   |
|  |  |  |  |  |  |  | ratio |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

What is the objective value? What is the basic feasible solution? Is the basis optimal?

The Simplex Tableau with respect to **B1 = { }:**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x1 | x2 | x3 | s1 | s2 | s3 | min |   |
|  |  |  |  |  |  |  | ratio |
|  |  |  |  |  |  |  |   |
|  |  |  |  |  |  |  |   |
|  |  |  |  |  |  |  |   |

What is the objective value? What is the basic feasible solution? Is the basis optimal?

3-3 (Continued if needed)

The Simplex Tableau with respect to **B = { }:**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x1 | x2 | x3 | s1 | s2 | s3 | min |   |
|  |  |  |  |  |  |  | ratio |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

What is the objective value? What is the basic feasible solution? Is the basis optimal?

The Simplex Tableau with respect to **B = { }:**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x1 | x2 | x3 | s1 | s2 | s3 | min |   |
|  |  |  |  |  |  |  | ratio |
|  |  |  |  |  |  |  |   |
|  |  |  |  |  |  |  |   |
|  |  |  |  |  |  |  |   |

What is the objective value? What is the basic feasible solution? Is the basis optimal?

4. (4 points) Calculate the inverse matrix *A*-1 of the following square matrix *A*. Show all your work. Your elementary row operations will be graded.

$$A=\left(\begin{matrix}1&2&3&4&5\\0&2&3&4&5\\0&0&3&4&5\\0&0&0&4&5\\0&0&0&0&5\end{matrix}\right)$$

5. (1 point) Transform the following LP into a standard form adding slack variables s1, s2 and s3. Note that x3 is unrestricted in sign. Remember Johnson’s pet peeve.

 Maximize 3 x1 + 5 x2 + x3

 Subject to – x1 – x2 + x3 = 0,

–2x1 + x3 ≤ 11,

 x3 ≤ 27,

 x1 + 4 x2 + x3 ≤ 90,

 x2 , x3 ≥ 0

 x1 unrestricted in sign.

6. (5 points) Consider the cutting stock problem to minimize the number of raws of 100 inches needed for

* 2000 finals of 12 in
* 3000 finals of 15 in
* 2500 finals of 20 in
* 2000 finals of 25 in

We are solving the problem by column generation approach. Let the initial basis use each roll the maximum number of times it can be cut from the raw. Model the knapsack problem to generate the column (or pattern) of the smallest reduced cost. Solve the problem generating entering columns by solving knapsack problems. Use ExcelSolver to price out. Work on Midterm2\_DIY\_CG4by4.xlsx and submit your work on the worksheet into the submission folder.