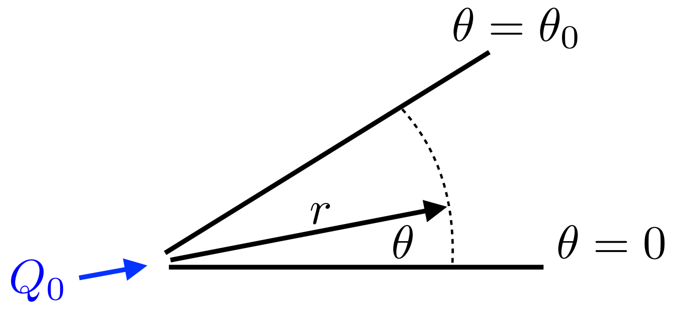
CHBE 501 - Exam #1

Question 2 - 30 points

(a) Consider a wedge formed by two flat plates that meet along the z-axis with an opening angle of θ_0 as shown. Where the plates meet, a small slit allows a volumetric



flow Q_0 to enter or leave the wedge per unit width in z. The flow is characterized by steady-state and fully-developed conditions in the absence of gravity and in which $\mathbf{v} = v_r\hat{\mathbf{r}}$, where $v_r(r,\theta)$ is a function only of r and θ. Assuming that $v_r(r,\theta) = f(r)g(\theta)$ can be written as a product of functions $f(r)$ and $g(\theta)$, write an integral expression in terms of v_r for the full volumetric flow (per width in z) $Q(r)$ through the surface defined at r (i.e., the dashed boundary shown). Assuming the fluid is incompressible, find $f(r)$ up to a multiplicative constant.

(b) In class, we calculated the steady-state, fully-developed laminar flow in a pipe of radius R, with the result that

$$v_z(r) = -\frac{\mathcal{P}'}{4\mu}\left(R^2 - r^2\right),$$

where $\mathcal{P}' = \frac{\partial}{\partial z}\mathcal{P}$ is the z-gradient of the dynamic pressure and μ is the viscosity of the fluid. What is the vorticity $\mathbf{w} = \nabla \times \mathbf{v}$?

(c) Consider planar flow of a Newtonian liquid of density ρ, viscosity μ and with $v_x = x^2 - y^2 + x$ and $v_y = -(2x+1)y$. Verify that the flow is incompressible. Is the flow irrotational? Find $\mathcal{P}(x,y)$, assuming that $\mathcal{P}(0,0) = 0$.