CHBE 501 - Exam #1

Question 3 - 20 points

(a) Show that the condition for the vectors **a**, **b**, and **c** to be coplanar is: $\varepsilon_{ijk} a_i b_j c_k = 0$.

(b) The Stokes theorem can be stated as

$$\int_C \mathbf{t} \cdot \mathbf{v} dC = \int_S \hat{\mathbf{n}} \cdot (\nabla \times \mathbf{v}) \, dS$$

for a vector field **v**. This is discussed in section A.5. Use this equality between surface (S) and bounding contour (C) integrals to prove that $\nabla \times (\nabla f) = 0$ for any single-valued twice-differentiable scalar f. Hint: consider various surface regions S and orientations $\hat{\mathbf{n}}$.