Analysing a simple system

A vehicle suspension system can be modelled by the block diagram shown in Figure 1 below:

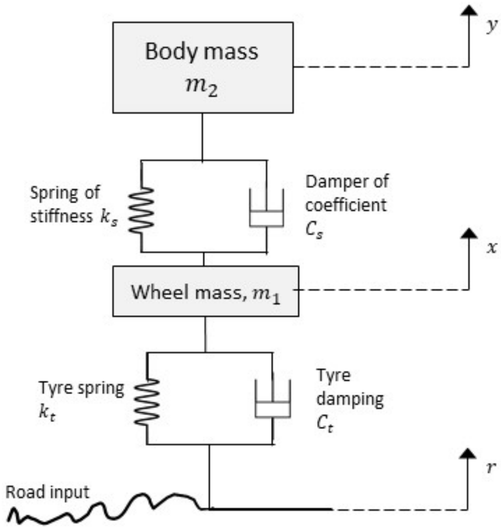

Figure 1: Block diagram of vehicle suspension system

In this block diagram, the variation in the road surface height r as the vehicle moves is the input to the system. The tyre is modelled by the spring and dashpot (damping) system with spring constant k_t and damping coefficient C_t respectively and this results in the displacement of the wheel (x), represented by the mass m_1. The wheel's displacement acts as an input to the suspension system, modelled by the spring and dashpot with spring constant k_s and damping coefficient C_s respectively and this results in the displacement (y), of the body, represented by the mass m_2. When the car is at rest, it is taken that $r = 0$, $x = 0$ and $y = 0$. (*Note*: m_2 is normally a quarter of the vehicle mass since it is assumed the weight is distributed evenly between the 4 wheels.

This system is composed of two mass-spring-damper systems 'stacked' one on top of the other. We shall first consider the behaviour of a single sub-system and then later attempt to combine these to find the overall system behaviour.

Consider the simple mass-spring damper system shown in Figure 2 below:

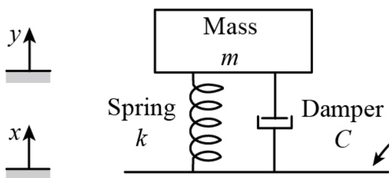

Figure 2: A single mass-spring-damper system

In Figure 2:

- *x* is the position of input body/surface, with its rest position given by *x* = 0.
- The mass *m* represents the mass
- The height of mass *m* above its reference level is called *y*. The reference level is chosen such that when system is at rest, *y* = 0.

Section 1: Mathematical Analysis of System (25 marks)

1. Draw a free-body diagram showing all the forces acting on the mass *m* shown in **Figure 2**.
2. From the earlier description, diagrams and the laws of Physics, **show that** the motion of the system in Figure 2 can be described by the LCCDE (linear constant-coefficient differential equation) below:

$$\frac{d^2y(t)}{dt^2} + \frac{C}{m}\frac{dy(t)}{dt} + \frac{k}{m}y(t) = \frac{C}{m}\frac{dx(t)}{dt} + \frac{k}{m}x(t) \qquad (1)$$

3. Using the Laplace transform of the equation above, find an expression for $H(s)$, the system transfer function.

The mass-spring-damper system is a damped second order system. It is common to express the homogenous second order DE for such a damped system as

$$\frac{d^2y(t)}{dt^2} + 2\zeta\omega_n\frac{dy(t)}{dt} + \omega_n^2 y(t) = 0 \qquad (2)$$

where ζ is the damping ratio and ω_n is the undamped natural (resonant) frequency.

4. From equations (1) and (2), determine expressions for ζ (the damping ratio) and ω_n (the natural frequency) in terms of the parameters *m*, *k* and *C*
5. Determine the characteristic equation and eigenvalues (characteristic values) for this system based on equation (2) above (in terms of ω_n and ζ).
6. From the answer to part 5, determine the full mathematical expression (in terms of ω_n and ζ) for the natural response of the system for the following cases:
    a. $\zeta = 0$
    b. $0 < \zeta < 1$
    c. $\zeta = 1$
    d. $\zeta > 1$

Consider a suspension system with the following parameters:

m_2 = 380 kg

k_s = 15,000 N/m

7. Determine ω_n (in rad/s) for this suspension system and the corresponding value for f_n (in Hz).
8. Calculate the required value of C_s in order to achieve $\zeta = 1$

Note: Complete and clear working is required for all answers for this section.

Section 2: System analysis using Matlab (30 Marks)

In this section, the system responses should be analysed using Matlab. Refer to the document *"A Brief MATLAB Guide"* in order to understand how to represent LTI systems in Matlab, and hence how to determine impulse response, step response and frequency response of systems. Sample Matlab code that has been provided as part of the learning materials can also be modified to suit. Students are advised to refer to the *help* function within Matlab as well as online Matlab documentation for more details.
Note: MATLAB is installed in the engineering computer labs and is also available to ECU students via ECU's site licence (refer to the announcement regarding site license for details).

Using the commands given in the *Guide*, analyse the response of the suspension system using the m_2 and k_s parameters given in Section 1 and C_s value calculated in question 8.

9. Plot the impulse response and step response of the system (for 2 seconds duration and time 'step size' of 1 millisecond) using the *impulse* and *step* functions. Include all plots (properly labelled) in your submission.

10. Determine the frequency response from 0 to 200 rad/s using the *freqs* command. Plot the magnitude and phase response over this frequency range. Hint: Use frequency 'step size' of 0.1 rad/s.

    Hint 1: You can plot all 4 graphs in one go using a 2 x 2 matrix of plots using *subplot*(22*n*), where *n* determines which of the 4 subplots gets used.

    Hint 2: In order to clearly see variations over a range of frequencies, it is best to use a log scale for the frequency and magnitude (phase would still be displayed using linear scale). The functions *loglog* (for magnitude) and *semilogx* (for phase) can be used instead of *plot*.

11. Determine the magnitude response at ω_n. Determine the frequency of the -3dB point (magnitude = $1/\sqrt{2}$ of passband). ***Hint:*** Use the 'data cursor' tool on the plot of the magnitude response. It shows the *x* and *y* values of the plot as you move along the curve.

12. Discuss the response of the system. Why do the impulse and step responses have that particular shape? How well will this system fulfil its purpose of a vehicle suspension?

    Note: The function of a suspension system is to 'filter out' the effect of bumps, potholes and other such road surface irregularities, but allow the vehicle to 'follow the road' as the height of the road surface varies.

13. Repeat the analysis above (steps 9 – 11) for the following damping ratios
    a. $\zeta = 0.4$
    b. $\zeta = 0.7$
    c. $\zeta = 1.5$
    d. $\zeta = 2.0$

    Hint 3: It would be more efficient to put all the necessary commands into a script file (a *.m* file) so you can edit the parameters and then run all the commands at once.

14. Based on the results of the Matlab analysis above, which of the 5 values of damping ratio would be best for application as a suspension system. Justify your selection.

Section 3: Modelling System response to input (25 marks)

The response of the suspension system can be modelled by providing an input *x* that resembles the road input (the assumption is that the tyre assembly does not impact the system response and therefore x is the same as road 'input signal' *r*).

The M-file for the custom-written function *inputsig* has been provided, with explanatory comments in the file.

15. Use the *inputsig* function to generate a sinusoid of 1 Hz with magnitude 0.5 and duration 2 seconds, then simulate the suspension system's response to this function and plot the input and output response vs time (output plot below input plot for easy comparison).

    Hint 4: The system response to an input signal can be simulated using the *lsim* function. Following is some sample code that shows how this function could be used.

    ```
    [xsig, t1] = inputsig(10);  % generate signal duration 10
    Sys1 = tf(B,A);              % define system, use tf (transfer function)
    Sysout = lsim(Sys1, xsig, t1);  % simulate LTI system response
    ```

    Note: Students are advised to refer to the *help* function within Matlab as well as the online Matlab documentation for more details.

16. Repeat step 15 (plots of input vs output) for input duration of **1 second** with the input signal being:
    a. a sinusoid of frequency 10 Hz and magnitude 0.05
    b. square wave of frequency 4 Hz and magnitude 0.1
    c. pulse of frequency 3 Hz and magnitude 0.05

17. Compare the system response to the 4 types of input and explain the output signal for each and the differences. What would these 4 represent in terms of 'road input'?

18. Repeat step 16 with a combination of:
    a. Sinusoid of 1Hz and magnitude 0.5 plus a sinusoid of frequency 10 Hz and magnitude 0.05
    b. Sinusoid of 1Hz and magnitude 0.5 plus a square wave of frequency 4 Hz and magnitude 0.1
    c. Sinusoid of 1Hz and magnitude 0.5 plus a pulse of frequency 3 Hz and magnitude 0.05
    d. Sinusoid of 1Hz and magnitude 0.5 plus a sinusoid of frequency 10 Hz and magnitude 0.1 AND a square wave of frequency 4 Hz and magnitude 0.1

19. Describe the differences in input and output waveforms in the cases in step 18. Hence comment on the effectiveness of the 'designed' suspension system.

Section 4: Including the wheel & tyre response in the analysis (20 marks)

The tyre and wheel assembly can also be modelled as a mass-spring-damper system (refer to Figure 1).

The parameters for a wheel and tyre system are as follows:

m_1 = 39 kg

k_t = 154,000 N/m

C_t = 10 Ns/m

20. Based on the parameters above, determine the natural frequency of this system ω_n
21. Find the impulse response, step response and frequency response of the system (same as in steps 9 – 11).
22. Compare the impulse and step responses to those for the suspension system. How does the tyre system response compare to what is generally observed in real life? Discuss any differences and reasons for them. (*Hint*: Refer to Tutorial 7 Question 7 solutions).
23. The frequency response of a 'cascade' system, such as this suspension and wheel combined system, can normally be worked out by combining the frequency response of the individual subsystems. Combine the frequency responses obtained in Sections 2 and 3 to find a predicted overall system frequency response (plots of combined magnitude and phase responses).
24. Discuss the overall frequency response found using this method and its validity.
25. Analysis of such suspension systems sometimes ignore the effect of the tyre's 'spring and damper' effect. Discuss the impact / validity of this approach based on your previous results and the parameter values involved.

~ End of Assignment Instructions ~