### Expand Question Completion Status:

### **QUESTION 1**

**Exercise 1**:   
Use a graph of (*x*−2)2=4sin(*x*) to find solutions to the equation valid to 2 decimal points:  
enter the roots...

**6 points**

### **QUESTION 2**

1. **Exercise 2**:   
   Use the zooming technique to find solutions of 50 + sin(x) = 2x  
   which are valid to at least two decimal places.   
   *Hint: Try to estimate the value of 50 + sinx. This will give you an idea in which x interval are the possible solutions!  
   enter a number...*

**6 points**

### **QUESTION 3**

**Exercise 3a**:   
Folklore is that exponential functions grow faster than polynomial functions. Although true, you need to be careful about how you interpret this statement, as this exercise shows.  
Consider the functions *z*1=*ex* and *z*2=*x*4. Plot them together on the interval [0,4].

* + From their graphs, how can you determine which graph is the exponential and which is the polynomial

|  |  |  |
| --- | --- | --- |
| a |  | a. Exponential functions grow faster than polynomial functions |
| b |  | B. For different values of x, I can evaluate z1, z2 and determine which is larger. |
| c |  | C. polynomial functions grow faster than exponential functions |

**6 points**

### **QUESTION 4**

1. **Exercise 3b:**Find the value of *x* (to two decimal places) for the point of intersection by zooming on the zero of *f*(*x*)=*ex*−*x*4. (or by zooming on the intersection point of the functions *z*1=*ex*, *z*2=*x*4.)

**6 points**

### **QUESTION 5**

1. **Exercise 3c:**On this graph, *x*4 is larger than *ex* from the intersection point to *x*=4. Experiment to determine how large a value of *x* is needed for the exponential to catch up to *x*4. Then find the  
   second intersection point. (correct to three decimal places.) This one is larger than 4. In fact, you now have found two intersection points (*x*1, *y*1), (*x*2, *y*2). (where *x*1 < *x*2) Up to *x*1 the  
   function *ex* is bigger, from *x*1 to *x*2 the function *x*4 is the bigger. What happens after *x*2?  
     
   What is the *x*-coordinate of the second intersection point?  
   enter a number...

**6 points**

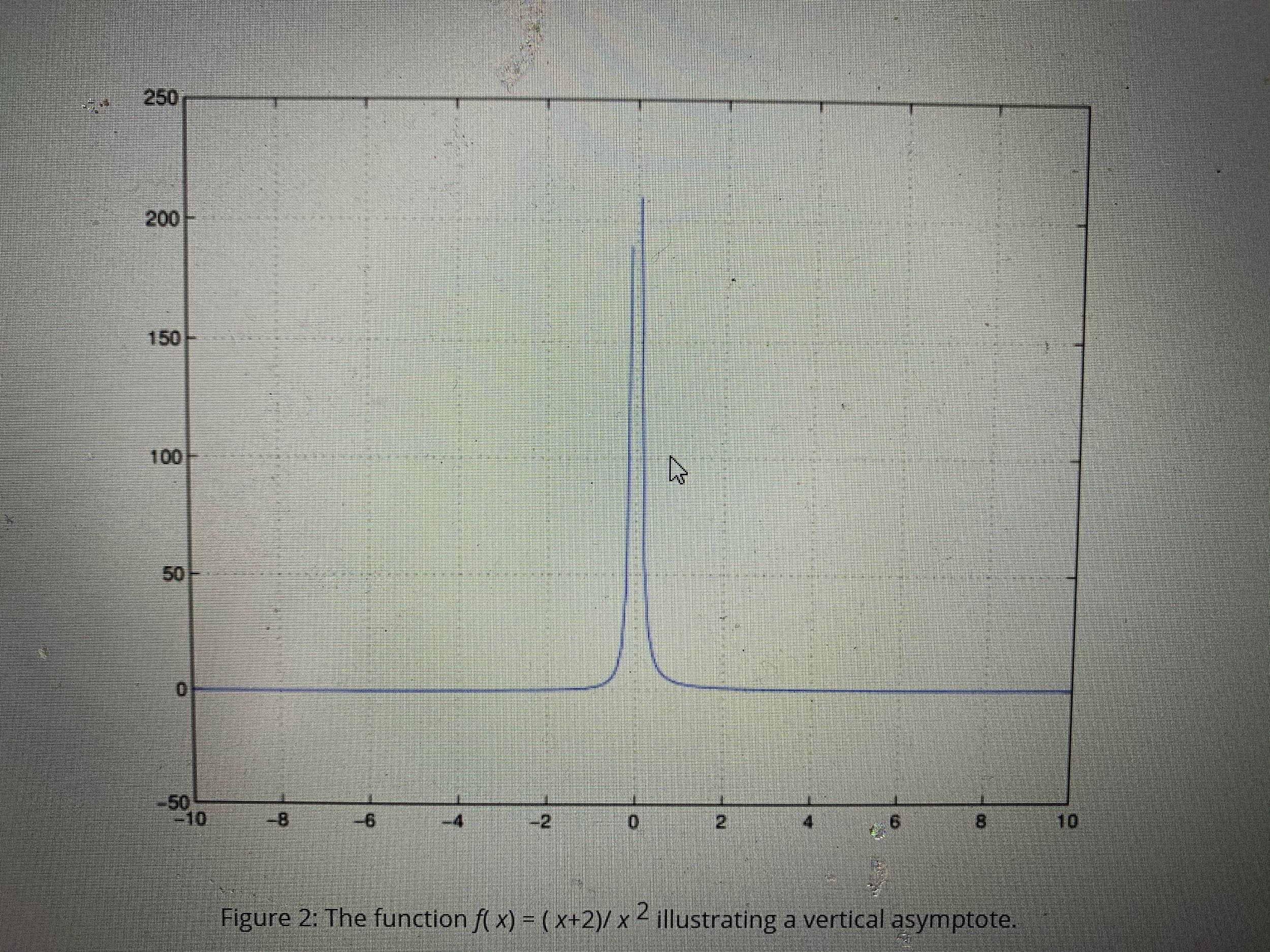
### **QUESTION 6**

1. Exercise 3d:  
   What happens to the behavior of *z*1 and *z*2 after the second intersection point?

|  |  |  |
| --- | --- | --- |
|  |  | 1. they grow at the same rate |
|  |  | 1. x4 grows faster they grow at the same rate |
|  |  | 1. ex grows faster, but for increasingly large values of x, x4 catches up to ex again. |
|  |  | 1. ex grows faster |

**6 points**

### **QUESTION 7**

1. Vertical asymptotes can be a real impediment to finding roots, as this example illustrates.  
   **Example 2**:   
   Find any zeroes and the minimum value of the rational function  
   f(x) = (x+2) / x2  
   We first graph the function over initial interval [−10,10] producing Figure .  
     
   Figure 2: The function *f*( *x*) = ( *x*+2)/ *x* 2 illustrating a vertical asymptote.  
   The figure illustrates that the vertical asymptote at *x*=0 gets in the way of finding the two answers we want-the zeroes and the minimum value. Let's try again using a little thought  
   before we chug ahead with the computer.  
   Recall that rational functions are 0 only when the numerator is, that is when *x*+2 = 0, or *x*=−2. The function goes from negative to positive there, so the minimum must be located to the  
   left of −2. As this function has a horizontal asymptote of *y*=0 there must be a minimum to the left of −2, and no minimum to the right of 0 where the function is positive. You'll do the  
   rest in the following exercise.  
   **Exercise 4a**:   
   * Find the *x*-coordinate for where *f*(*x*)=(*x* + 2)/*x*2 achieves its minimum value.

**6 points**

### **QUESTION 8**

1. **Exercise 4b:**What sub-interval along the *x*-axis makes for a plot window diplsaying the most detail?   
     
   Which linspace is best?

|  |  |  |
| --- | --- | --- |
|  |  | 1. x=linspace(-10,-2); |
|  |  | 1. x=linspace(-5,1); |
|  |  | 1. x=linspace(-5,-3); |
|  |  | 1. x=linspace(-5,3); |

**6 points**

### **QUESTION 9**

1. **3 Finding roots of polynomials**Polynomial functions are extensions of linear and quadratic functions : (a≠0)  
     
   f(x) = ax+ b(linear), f(x) = ax2 + bx + c (quadratic)

For these two types of functions we can solve *f*(*x*) = 0 easily. For the linear case the single solution is −*b*/*a*. For the quadratic case the quadratic formula produces two answers. These answers are the *roots* of the polynomial.  
The quadratic formula produces two roots, although many possible cases exist: two distinct real roots, double roots, or two complex-valued roots that come in conjugate pairs. There are always no more than 2 real roots.  
A general polynomial of degree *n* may be written as  
f(x) =anxn + 1xn-1+ … + a2x2 + a1x1 + a0  
  
For an *n*th degree polynomial there are *n*, possibly complex, roots counting multiplicities. But they need not all be real roots. However, there is not always a formula for solving for these roots.  
However, MATLAB has a built-in function, called roots, that numerically solves for the roots, which is faster and more accurate than the zooming technique. To use roots, you must code the polynomial *f*(*x*) like a vector. We take each coefficient (there are *n*+1) and enter them in order as follows:  
  
The polynomial *f*( *x*) is represented with  
>> p = [ an an-1 … a2  a1 a0]  
  
Then the polynomial equation *f*(*x*)=0 is solved with the command  
>> roots(p)   
  
 The only subtlety is that we don't usually write the terms whose coefficients are 0, but we must do so when entering in the polynomial into MATLAB. So, for instance

[3 2 1] represents f(x) = 3x2 + 2x + 1,

[3 2 1 0] represents f(x) = 3x3 + 2x2+ x,

[3 2 0 1] represents f(x) = 3x3 + 2x2 + 1, and

[3 0 0 2 1] represents f(x) = 3x4 + 2x + 1.  
  
**Example 3**:   
Use the roots function to find the roots of *f*(*x*)=2*x*3+6*x*2−4*x*−5.  
>> p = [2 6 -4 -5] p = 2 6 -4 -5 >> roots(p) ans = -3.3732 1.0675 -0.6943   
  
 We get 3 real roots. We could verify this by plotting the polynomial over the interval [−4,2] and observing the graph crosses the *x* axis three times. Instead, we illustrate that these are roots, by evaluating the polynomial at these values:  
>> x = roots(p) % store the values  
>> 2\*x.^3 + 6\*x.^2 -4\*x -5 % evaluate the polynomial for roots  
 ans = 1.0e-13 \* -0.3197 0.0444 0   
 These values are *very close* to 0, as they are multiplied by 1.0e-13. This is MATLAB's scientific notation for 10−13.  
**Exercise 5a:**

* + Let *f*(*x*)=*x*3−7*x*2+2*x*+9. Solve the cubic equation *f*(*x*)=0. Find all of its roots correctly up to 4 significant digits.

|  |  |  |
| --- | --- | --- |
|  |  | 1. 6.6 , 1.1 -0.7 |
|  |  | 1. 6.4766, 1.4692, -0.9458 |
|  |  | 1. 0.0010, 1.0100, 7.5902 |
|  |  | 1. 6.5806 , 1.1062, -0.6868 |
|  |  | 1. 6.7053 , 1.3259 , -0.8259 |

**6 points**

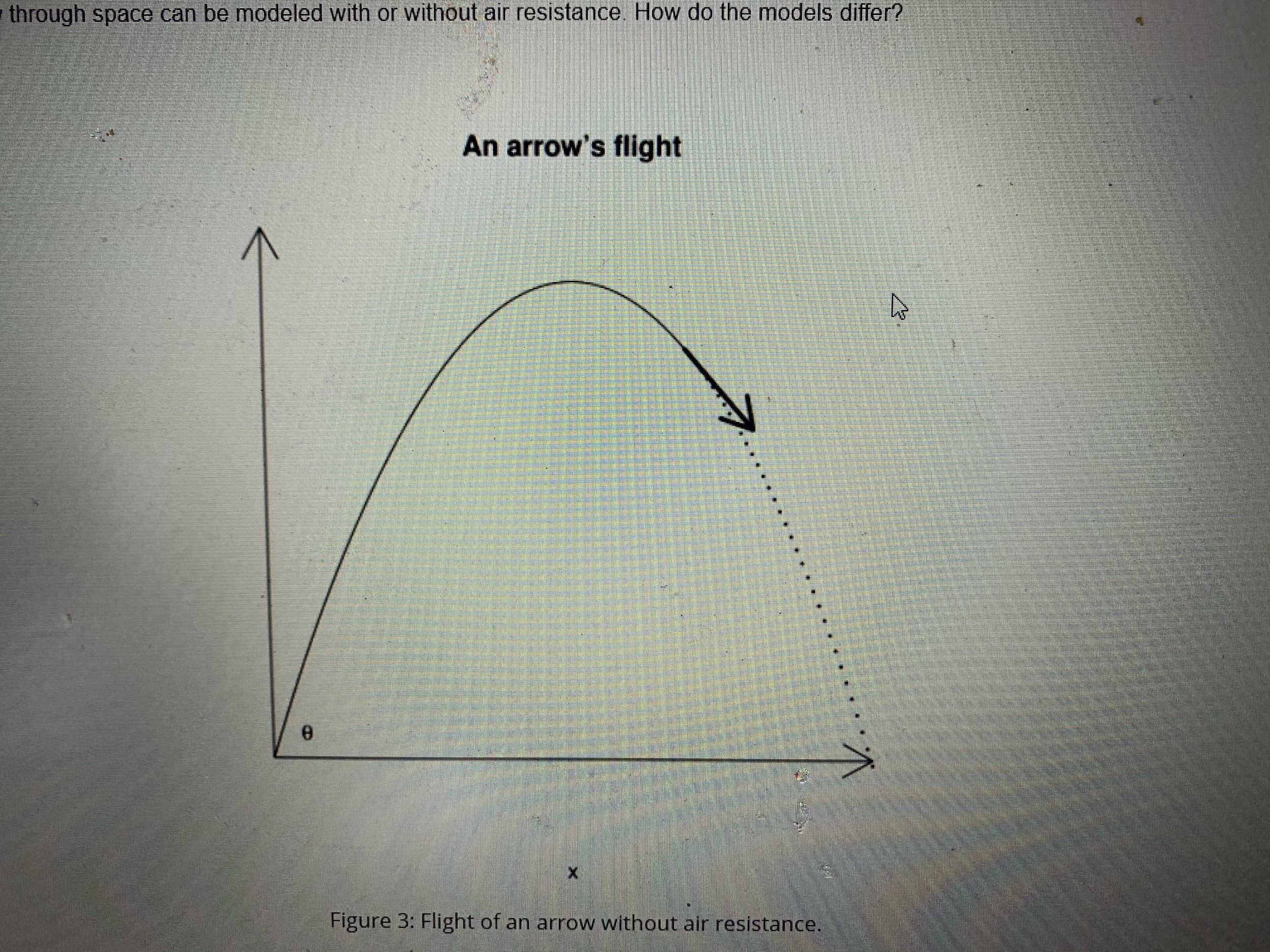
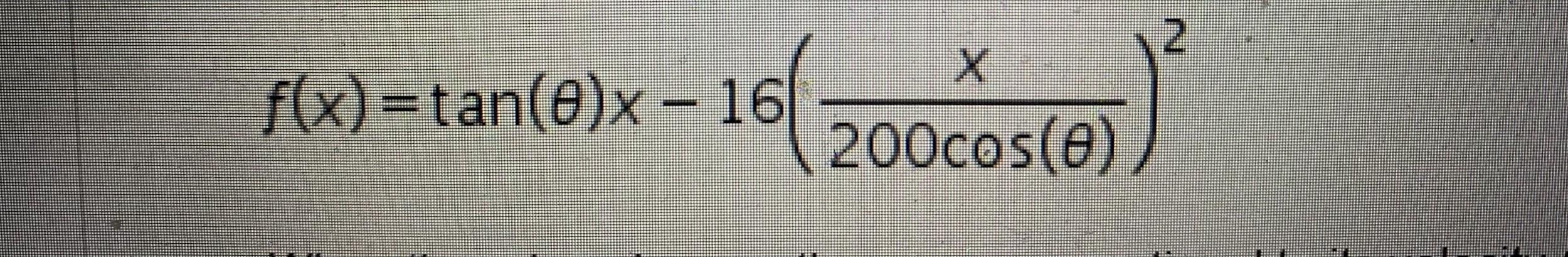
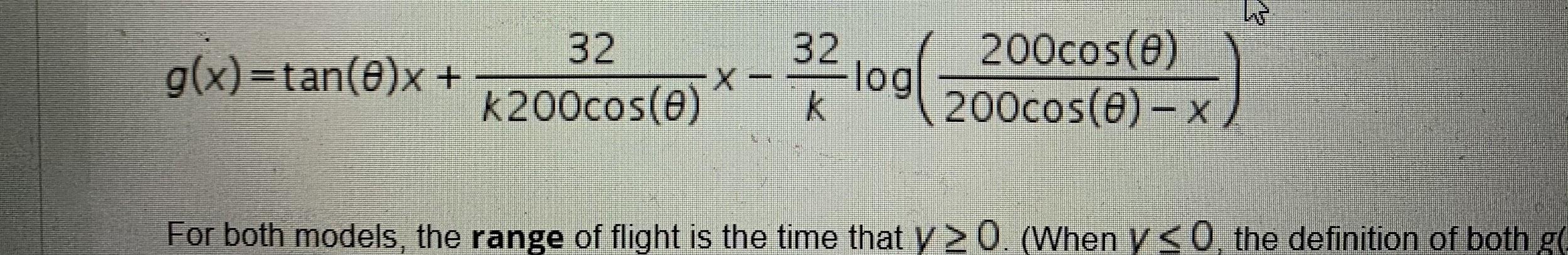
### **QUESTION 10**

**Exercise 5b:**Now find all solutions to *x*3+2*x*+4=0 (Note that the coefficient of *x*2 is now 0).

|  |  |  |
| --- | --- | --- |
|  |  | 1. 1.8230, -1.8230, -1.3283 |
|  |  | 1. 0.6641, -0.6640, -1.3283 |
|  |  | 1. 0.5898 ±1.7445i, −1.1795 |
|  |  | 1. 1.8230 ±0.6641i , -1.3283 |

**6 points**

### **QUESTION 11**

1. This last exercise illustrates that both means of finding zeroes of functions can be useful.  
   **Example 4**:   
   The flight of an arrow through space can be modeled with or without air resistance. How do the models differ?  
     
   Figure 3: Flight of an arrow without air resistance.  
   When an arrow encounters no air resistance the laws of projectile motion from high-school physics apply. Using *x* for the horizontal distance traveled, *f*(*x*) for the height of the arrow when it is *x* units away, and θ for the initial angle, we have this model for the trajectory of an arrow without air resistance:  
     
   When there is a drag on the arrow proportional to its velocity (*k* is the proportion factor), the height of the arrow is given by:  
     
     
   For both models, the **range** of flight is the time that y ≥ 0. (When y ≤ 0, the definition of both *g*(*x*) and *f*(*x*) should be set to 0.) The range can be written as [0,*b*].   
     
   First, we investigate an arrow's flight without wind resistance.  
   **Exercise 6**:   
   * Let 𝜃= π/4 . Look carefully at *f*(*x*), it is a quadratic polynomial in *x*. Rewrite *f*(*x*) so that the coefficients appear as (careful with the scientific notation)
   * f(x) = ax2 + bx + c  
      What are the values of a, b and c?  
     type the three values in decimal form, separated by commas

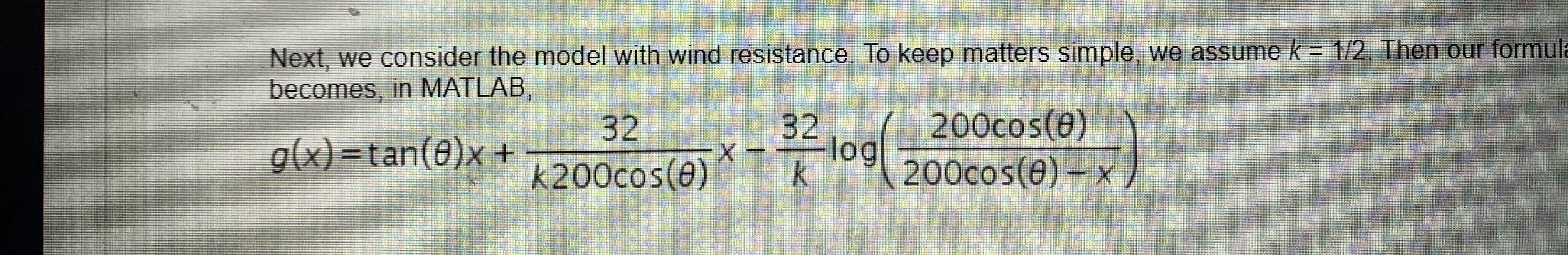
**6 points**

### **QUESTION 12**

1. **Exercise 6b:**Use your previous answer and the *roots* function to find the range of the arrow when shot at an angle of 𝜃= π/4  
   (By range we mean the horizontal distance between where the arrow was launched, i.e., the origin, to the spot it lands on the ground --  
     
   the first positive x-intercept of the function.) This can be accomplished by use of the roots command, and confirmed by visual inspection of the graph.

**6 points**

### **QUESTION 13**

1. Next, we consider the model with wind resistance. To keep matters simple, we assume *k* = 1/2. Then our formula for the trajectory when θ = π/4 becomes, in MATLAB,  
     
     
   >> theta = pi/4;  
   >> k = 1/2 ;  
   >> a = 200\*cos(theta);  
   >> b = 32/k;  
   >> g = (tan(theta)+ b/a)\*x - b\*log(a ./(a-x));  
     
   *provided* you have defined values for x.  
   **Exercise 7a**:   
   * + Plot various graphs of the *g*(*x*) until you find the range of *g*. Enter this value with at least 1 digit to the right of the decimal point.  
       (Remember, arrows don't bounce up - this mathematical model is only valid until the arrow first hits the ground.)

**6 points**

### **QUESTION 14**

1. **Exercise 7b:**Now make a plot containing the trajectories of both models. Label the individual plots.
   1. Attach File

**10 points**

### **QUESTION 15**

1. **Exercise 7c:**From your graph estimate the maximum height of the arrow if there is no wind resistance.

**6 points**

### **QUESTION 16**

1. **Exercise 7d:**From your graph estimate the maximum height of the arrow if there **is** wind resistance.

**6 points**