CSCI/MATH2308.1 2020

Assignments should be submitted to the appropriately titled Brightspace Dropbox. For computer problems, make sure to submit a copy of the source code you have written and as well as a copy of the output produced by your program. All computer programs should be written in Fortran 95.

1 Consider the two mathematically equivalent expressions

$$f(x) = x(\sqrt{x+1} - \sqrt{x}), \quad \text{and} \quad g(x) = \frac{x}{(\sqrt{x+1} + \sqrt{x})}$$

a) Prove that the two expressions are mathematically equivalent.

b) Which of the two expressions can be evaluated more accurately in floating point arithmetic? Why?

c) Using 4-digit precision, at $x = 500$, $f(500) = 10$, and $g(500) = 11.17$. Which of these two evaluations is correct? Explain the discrepancy by performing the computation and analyzing the interim steps. Use an arbitrary precision calculator such as https://apfloat.appspot.com/.

2 Assume that you are solving the quadratic equation $ax^2 + bx + c = 0$, with $a = 1.22$, $b = 3.34$, and $c = 2.28$, using the standard quadratic formula, and using 3-digit arithmetic with rounding. Use an arbitrary precision calculator such as https://apfloat.appspot.com/.

a. What is the computed value for $b^2 - 4ac$?

b. What is the exact value for $b^2 - 4ac$?

c. What is the relative error for the computed value for $b^2 - 4ac$?

3 When a and b are the same sign, which of the following two formulas is preferable for computing the midpoint m of an interval $[a, b]$, in **floating point arithmetic** with rounding? Why? [Hint: It is possible to devise example(s) in which the midpoint lies **outside** the interval $[a, b]$, for **only one** the formulas above.]

$$m = \frac{a+b}{2} \qquad or \qquad m = a + \frac{b-a}{2}$$

Programming Questions

1)

Write a program to compute the mathematical constant e, the base of natural logarithms from the definition

$$e = \lim_{n\to\infty}\left(1.0 + \frac{1.0}{n}\right)^n.$$

Specifically, compute $\left(1.0 + \frac{1.0}{n}\right)^n$ for $n = 10^k$, $k = 1, 2, \ldots, 20$. Determine the error in your successive approximations by comparing them with the exact value, e. Does the error always decrease as n increases? Explain your results.

2)

a) Consider the function

$$f(x) = \frac{(e^x - 1)}{x}.$$

Use L' Hospital's rule to show that

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\frac{(e^x - 1)}{x} = 1.$$

b) Check this result empirically by writing a program to compute $f(x)$ for $x = 10.0^{-k}$, $k = 1, \ldots\ldots\ldots.., 15$. Do your results agree with theoretical expectations? Explain why.

3)

Write a program to compute the absolute and relative errors in Stirling's approximation

$$n! \approx \sqrt{2\pi n}\ \left({}^{n}/_{e}\right)^n.$$

for $n = 1, 2, \ldots\ldots\ldots, 10$. Does the absolute error grow or shrink as n increases? Does the relative error grow or shrink as n increases?