Experiment 7
Centripetal Force

Questions

If an object moves in a circle and the centripetal force "turns off" what happens? How are centripetal and centrifugal forces related? How do you propagate errors through an equation that has three terms? How do you compare two numbers when each has an error margin?

Concepts

Newton's Third Law is the *action-reaction* law. Whenever something exerts a force on an object, the object exerts a *reaction* force on that something. The reaction force points in the opposite direction to the action force and it is equal in magnitude to the action force. Remember, the action and reaction forces act on *different* objects. A rocket goes up because of the upward thrust of the expanding exhaust gasses acting on the engine's nozzle. The nozzle exerts a force on the exhaust gasses that points down. The forces have the same magnitude but they act on different objects. Newton's Third Law is written as:

$$\vec{F}_{\text{on A by B}} = -\vec{F}_{\text{on B by A}} \tag{1}$$

Suppose you tie a rock to the end of a string and swing it in a horizontal circle over your head. A horizontal circle ensures that gravity does not change the angular speed of the rock. If you release the string, the rock flies out along a straight-line path, tangent to the circle. This suggests that the rock's velocity vector was tangent to the circle just before release.

A constant rate of rotation is called *uniform circular motion* UCM. Here, the object revolves in a constant period of time, T. The length of the velocity vector is constant. The direction of the velocity vector *changes* with time. Since velocity is a vector, any change in that vector (even in direction) represents an acceleration. This acceleration is always directed toward the center of the circle. It is called a *centripetal* acceleration, which means *center seeking*.

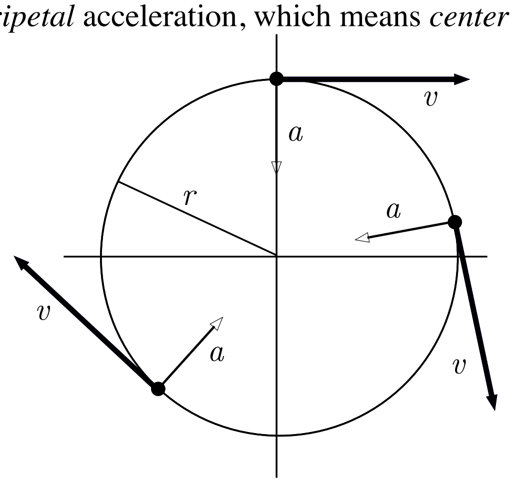

Figure 1. The velocity and acceleration vectors for uniform circular motion.

From Newton's Second Law, whenever a net force acts on an object it accelerates. Not surprisingly, the force that produces the object's centripetal acceleration is called the centripetal force. In the example of the rock rotating over your head, the centripetal force acts on the rock by the string. Physically, it is the tension in the string. Another example is when you drive a car around a circular curve. The centripetal force is a center-directed frictional force that acts on the car tires by the road. Normally, this force is large enough to keep the car in its lane of traffic. If this force is not large enough (or if the car's speed is too great) the radius must be larger to compensate and the car slides outward into the other lane.

The term *centrifugal*, which means *away from center* is sometimes confusing. The standard notion is that another force acts on the object moving in a circle and this force is directed outward from center. This notion is NOT correct. The centrifugal force is actually the Newton's Third Law reaction force to the centripetal force. Recall you obtain a reaction force by switching the labels "**A**" and "**B**" if you use the $\mathbf{F}_{\text{on }\mathbf{A}\text{ by }\mathbf{B}}$ convention for describing a force **F**. In the two examples mentioned previously:

EXAMPLE	CENTRIPETAL	CENTRIFUGAL
rock tied to string	$\mathbf{F}_{\text{on }\mathbf{rock}\text{ by }\mathbf{string}}$	$\mathbf{F}_{\text{on }\mathbf{string}\text{ by }\mathbf{rock}}$
car rounding a turn	$\mathbf{F}_{\text{on }\mathbf{tire}\text{ by }\mathbf{road}}$	$\mathbf{F}_{\text{on }\mathbf{road}\text{ by }\mathbf{tire}}$

As you can see, the centrifugal force does NOT act on the object of interest that is moving in a circle (the rock and the tire). We can instead shift our focus to the string as the object moving in a circle. Your hand holds the string and this is why you feel a pull outward. The force on your hand by the string is directed outward and the force on the string by your hand is directed inward. Once again, the object moving in a circle (now the string) experiences a centripetal force and NOT a centrifugal force.

Another example is when you feel pressed against the interior of a car door when the car takes a corner. Suppose you were traveling in a straight line. Newton's First Law dictates you will tend to continue in that path. However, the car turns and the car door intercepts your straight-line path. If a force (directed toward the center of the rotation) acts on you, you will remain upright in the car seat. This is the centripetal force provided by the car door.

Method

The centripetal force apparatus consists of a heavy metal bob connected by a spring to a central, rotating shaft. See Fig. 2. The shaft is turned by hand. As the bob sweeps out a circular path, it stretches the spring. Were it not for the spring, the bob would tend to fly off in a straight-line tangent to the circle. Once stretched, the spring will supply a centripetal force, which is directed toward the center of the circle.

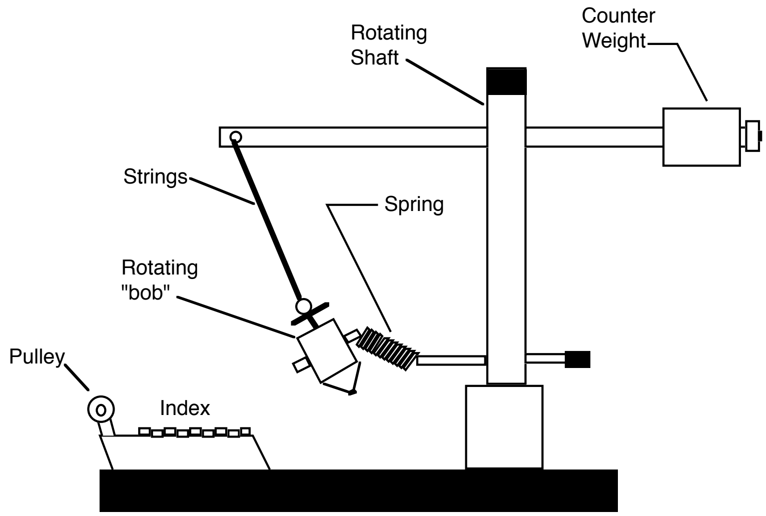

Figure 2. The Beck Centripetal Force Apparatus

If the bob moves at a uniform speed, v, in a circle of constant radius, r, this force will also be of constant magnitude and equal to the mass of the object multiplied by its centripetal acceleration:

$$F = \frac{mv^2}{r} \tag{4}$$

In this experiment, you calculate the centripetal force exerted by the spring on the bob. The calculation is checked by directly measuring the spring's tension.

Procedure

1) Derive the equation you will use to calculate the centripetal force from the mass of the bob, m, the number of revolutions counted, N, and the frequency of revolutions, f. Check with your instructor to verify you have the correct equation before continuing. Include this derivation in the data analysis section of your report.

2) Adjust where the cross-bar is clamped to the rotating shaft so the un-stretched spring pulls the bob about 2 - 3 cm away from vertical. Disconnect the spring and record the radius of rotation from the index.

3) Reconnect the spring and rotate the shaft fast enough so the bob continuously maintains this constant radius of rotation. Continually exert a small torque on the top of the rotating shaft to keep the bob's radius as constant as possible.

4) Examine the bob as it flies past the chosen index marker. Estimate the variation in the radius of the bob's circular path. If the bob swings back and forth through a small distance of b centimeters, then the error in the radius is ± $b/2$. Record this error (in meters) on your data sheet. Calculate the percent error in the radius.

5) Measure the time T it takes for the bob to make N complete revolutions (you decide on N). Record N and T then calculate (in a spreadsheet) the frequency f. Make more timings and calculate the average frequency and its sample standard deviation.

6) The error in the radius is not easily controlled. More than likely, the percent error in the radius will be around 2% or larger. You *can* control the percent error in the frequency with additional timings. Therefore, make additional timings until the percent error in the frequency is significantly less than the percent error in the radius. *Significantly less* can be half since addition in quadrature shows $\sqrt{x^2 + \left(0.5x\right)^2} \approx x$

7) Measure the bob's mass and estimate an error in this value. Then, calculate the percent error in the bob's mass.

8) Slide a paper clip into the small hole drilled in the bob. Tie a length of string to the paper clip and run it over the pulley. Tie a loop in the other end of the string and hang a weight hanger from it. Add mass to the hanger until the bob hangs vertically. Record the mass and calculate the static equivalent centripetal force. Note: The angled weight hangers are not exactly 50 grams, so measure their mass using a triple beam balance.

9) Derive an expression for the propagated error in the centripetal force calculated from m, r and f. See Appendix C. Check with your instructor to verify you have the correct equation before continuing. Include this derivation in the data analysis section of your report.

10) In a spreadsheet, calculate the experimental value for the mass multiplied by the acceleration, which is equal to the centripetal force. Also, use your error propagation equation to find the percent error in this calculated centripetal force. Then, convert this percent error into an absolute error.

11) Calculate the difference (or discrepancy) d between your best calculated centripetal force and the static measurement of the same quantity. Also calculate the propagated error in this difference, σ_d (see Appendix C). Organize all calculations in a spreadsheet. Include the spreadsheet in your report.

12) ***Question:*** Do your two values agree within the limits of experimental error? If not, explain the discrepancy. Hint: There is a systematic error overlooked in the theory. Cite specific factors and analyze how a specific, overlooked error would affect the result.