**Principles of Investment**

**Learning Exercise**

**Diversification**

1. **Assume that an individual can either invest all of his resources in one of the two securities, A or B; or, alternatively, he can diversify his investment between the two. The distributions of the returns are as follows:**

|  |  |
| --- | --- |
| **Security A** | **Security B** |
| **Return** | **Probability** | **Return** | **Probability** |
| **-10%** | **0.5** | **-20%** | **0.5** |
| **50%** | **0.5** | **60%** | **0.5** |

**Assume that the correlation between the returns from the two securities is zero, and answer the following questions:**

1. **Calculate each security's expected return, variance and standard deviation.**
2. **Calculate the probability distribution of the returns on a mixed portfolio comprised of equal proportions of securities A and B, i.e. calculate all possible returns on this portfolio and the probability of each one.[[1]](#footnote-1)**
3. **Also calculate the portfolio's expected return, variance and standard deviation.**
4. **Calculate the expected return and the variance of a mixed portfolio comprised of 75% of security A and 25% of security B.**
5. **The securities of companies Z and Y have the following expected returns and standard deviations:**

|  |  |  |
| --- | --- | --- |
|  | $$E\left[r\_{i}\right]$$ | $$σ\_{i}$$ |
| **Company Z** | **15%** | **20%** |
| **Company Y** | **35%** | **40%** |

**Assume that the correlation between the returns of the two securities is 0.25.**

 **(a) Calculate the expected return and standard deviation for the following portfolios:**

**(1) 100%Z**

**(2) 75%Z + 25%Y**

**(3) 50%Z + 50%Y**

**(4) 25%X + 75%Y**

**(5) 100%Y**

**(b) Graph your results (standard deviation on the X axis, Expected return on the Y axis).**

**(c) Which of the portfolios in part (a) is not optimal? Explain.**

1. **Consider two assets A and B for which return distributions can be summarized as follows:**

|  |  |  |
| --- | --- | --- |
|  | $$E\left[r\_{i}\right]$$ | $$σ\_{i}$$ |
| **Asset A** | **3%** | **1%** |
| **Asset B** | **7%** | **2%** |
| **Correlation Coefficient** | $$ρ\_{AB}=0$$ |

**What is the risk of the minimum risk portfolio composed of these two Stocks? (Hint: Use the calculus to minimize σp2). Is the risk of the minimum risk portfolio below that of every constituent asset? What is the expected ROR on the minimum risk portfolio?**

1. **Consider two other assets A’ and B’, which are identical (in statistical summary), respectively, to A and B above except that ρAB = 1.**
2. **Draw the graph of the efficient frontier in this case**
3. **Write down the answers to the same questions as in problem 3.**
4. **Assume N securities. The expected returns on all the securities are equal to 0.01 and the variances of their returns are all equal to 0.01. The covariances of the returns between two securities are all equal to 0.005.**
	1. **What are the expected return and the variance of the return on an equally weighted portfolio of all N securities? Please, note that the variance is presented by the formula, which depends on N.**
	2. **What value will the variance approach as N gets large?**
	3. **From parts (i) and (ii): Can you conclude what characteristic of the securities is most important when determining the variance of a well-diversified portfolio?**
5. **The returns on stocks A and B are perfectly negatively correlated (). Stock A has an expected return of 21 % and a standard deviation of return of 40%. Stock B has a standard deviation of return of 20%. The risk-free rate of interest is 11 %. What must be the expected return to stock B?**
6. **Consider the situation of an insurance company which offers personal injury policies to professional hockey players. The typical payoffs to one of these policies are forecast to be the following:**

|  |  |  |
| --- | --- | --- |
| **State** | **Prob.**  | **Payoff** |
| **Athlete is seriously injured** | **0.01** | **-5,000,000** |
| **Athlete is not injured seriously** | **0.99** | **60,000** |

**What is the expected profit per policy? Assuming the returns are uncorrelated policy to policy, how many policies must be sold in order for the standard deviation of the firm's overall portfolio of injury policies to be below $10,000?**

1. In the case of the two independently distributed returns the joint probability that the return on A is x% and the return on B is y% at the same time is the product of marginal probabilities. That is

$$prob\left(r\_{a}=x and r\_{b}=y\right)=prob\left(r\_{a}=x\right)×prob\left(r\_{b}=y\right)$$

 [↑](#footnote-ref-1)