Intermediate Statistical Theory

Homework Sheet 4

Basic Questions

1. Show that the sample proportion $\frac{X}{n}$ is a minimum variance unbiased estimator of the binomial parameter θ (Hint: Treat $\frac{X}{n}$ as the mean of a random sample of size n from a Bernoulli population with the parameter θ.)

2. If $\bar{X}_1$ is the mean of a random sample of size n from a normal population with the mean μ and the variance σ_1^2, $\bar{X}_2$ is the mean of a random sample of size n from a normal population with the mean μ and the variance σ_2^2, and the two samples are independent, show that

   (a) $\omega\bar{X}_1 + (1 - \omega)\bar{X}_2$, where $0 \le \omega \le 1$, is an unbiased estimator of μ; (b) the variance of this estimator is a minimum when $\omega = \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}$.

   (c) find the efficiency of the estimator of part (a) with $\omega = 1/2$ relative to this estimator with $\omega = \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}$.

3. Let $Y_1, Y_2, \ldots, Y_n$ be a random sample of size n from the pdf

$$f_Y(y;\theta) = \frac{1}{(r-1)!\theta^r} y^{r-1} e^{-y/\theta}, y > 0$$

   (a) Show that $\hat{\theta} = \frac{1}{r}\bar{Y}$ is an unbiased estimator for θ.

   (b) Show that $\hat{\theta} = \frac{1}{r}\bar{Y}$ is a minimum-variance estimator for θ.

4. Let X_n denote a random variable with mean μ and variance b/n^p, where $p > 0$, μ and b are constants (not functions of n). prove that X_n converges in probability to μ. (use Chebyshev's inequality).

5. An estimator $\hat{\theta}_n$ is said to be *squared-error consistent* for θ if $\lim_{n\to\infty} E[(\hat{\theta}_n - \theta)^2] = 0$.

   (a) Show that any squared-error consistent $\hat{\theta}_n$ is asymptotically unbiased.

   (b) Show that any squared-error consistent $\hat{\theta}_n$ is consistent.