1. Let $X$ be the set of $2 \times 2$ matrices. Define

$$d : X \times X \to \mathbb{R}; \quad d(x, y) = \max_{1 \le i,j \le 2} \{|x_{i,j} - y_{i,j}|\}$$

where $x_{i,j}$ is the $i, j$-th entry of $x$. Prove or disprove that this is a metric on $X$.

2. Let $A = \left\{ \frac{2}{n} \ : \ n \in \mathbb{Z} \right\}$. Does $A$ have a supremum and/or an infimum? If so, prove it.

3. Let $A = \left\{ \frac{2}{n} \ : \ n \in \mathbb{Z} \right\}$. Determine if $A$ is a closed set. Prove or disprove from the definition of a closed set.

4. Let $X$ be a set. Define

$$d : X \times X \to \{0, 1\} \subseteq \mathbb{R}$$
$$d(x, y) = \begin{cases} 0 & x = y \\ 1 & x \neq y \end{cases}.$$

Let $x_0 \in X$, consider the open ball of radius 1 around $x_0$,

$$B(x_0, 1) = \{x \in X \ : \ d(x, x_0) < 1\}$$

the closed ball of radius 1 around $x_0$,

$$\bar{B}(x_0, 1) = \{x \in X \ : \ d(x, x_0) \le 1\}$$

and the closure of the open ball of radius 1 around $x_0$, denoted $\overline{B(x_0, 1)}$. Determine, explicit descriptions of these sets and prove your answers.

**Hint:** It is not true in general that the closure of the open ball and the closed ball are the same set!
**Double Hint:** Is a singleton set $\{x_0\}$ open or closed or both with this metric?