ECONOMICS - Part II

Coursework Assignment: ECON 325 - Advanced Microeconomics

Instructions:

- You should answer **all questions** and submit your answers electronically (in **pdf format**) through the Moodle site for this course.
- Please note that you will have to submit **two pdf files** (one for each section of the worksheet).
- Your submission can be a good-quality scanned copy (or picture) of your hand-written solutions.
- **Assignment due Friday, 24 April 2020, 17:00 (British Summer Time).**

Section A

1. **[25 marks]** Consider a pure-exchange, private-ownership economy, consisting of two consumers A and B, who trade two commodities, denoted by $l = 1, 2$. Initial endowments are $\mathbf{R}^A = (2, 8)$ for consumer A and $\mathbf{R}^B = (8, 2)$ for consumer B. Individual utility functions are $U^A(x_1^A, x_2^A) = (x_1^A)^{(1/2)}(x_2^A)^{(1/2)}$ and $U^B(x_1^B, x_2^B) = \ln(x_1^B) + \ln(x_2^B)$.

   (a) Find the equations describing the contract curve and the 2-person core. Is the allocation $[\underline{\mathbf{x}}] = [(\underline{x}_1^A, \underline{x}_2^A), (\underline{x}_1^B, \underline{x}_2^B)] = [(4, 4), (6, 6)]$ part of the 2-person core? Justify your answer.

   [10 marks]

   (b) Assume the number of agents in this economy is doubled by balanced replication from 2 to 4. Find an allocation $[\mathbf{x}^+]$ that can be used by the coalition $\{A, A, B\}$ to block $[\underline{\mathbf{x}}]$. Using this result, what can you infer about the relationship between the size of the population and the size of the core?

   [10 marks]

   (c) Assume the economy is replicated by a factor $N \to \infty$. Derive the core allocation(s)?

   [5 marks]

2. **[25 marks]** In an economy there are many identical price-taking firms, using a constant returns to scale technology, in which labor is the only input. The price of every firm's output is 1 such that the profit per worker is $\theta - w$, where θ is the worker's productivity (i.e. the number of units of output this worker can produce) and w is the wage.

   There are two types of workers: type-a workers have productivity $\theta_a = 3$ and type-b workers have productivity $\theta_b = 1$. Workers' productivities are unobservable by firms but they know that the proportion of type-b workers is π. Both types of workers have a reservation wage of zero.

   (a) What is the equilibrium wage set by the firms? What types of workers are going to accept an employment offer?

   [5 marks]

   Suppose workers can spend their own resources to acquire educational certificates in order to signal their productivity. It is common knowledge that the cost of acquiring an education level z equals z^2 for type-b workers and $\frac{z^2}{2}$ for type-a workers.

   (b) Characterize the set of separating Perfect Bayesian equilibria. For what values of π will the type-a and type-b workers be better off under the no-signalling outcome in part (a) than under the least-cost separating equilibrium?

   [10 marks]

   Suppose now that workers don't have access to educational certificates, but firms create jobs with different task levels $t \geq 0$. Let the proportion of type-b workers be $\pi = \frac{3}{4}$. The output produced by a worker of type i is θ_i $(i = a, b)$ regardless of the worker's task level, i.e. t is used as a screening mechanism and does not affect profit directly. Task levels affect workers' utility: $u_a = w - \frac{1}{4}t^2$ and $u_b = w - \frac{1}{2}t^2$. In stage 1, firms simultaneously announce a menu of contracts (w, t). In stage 2, workers decide whether they want to sign a contract and which one to sign:

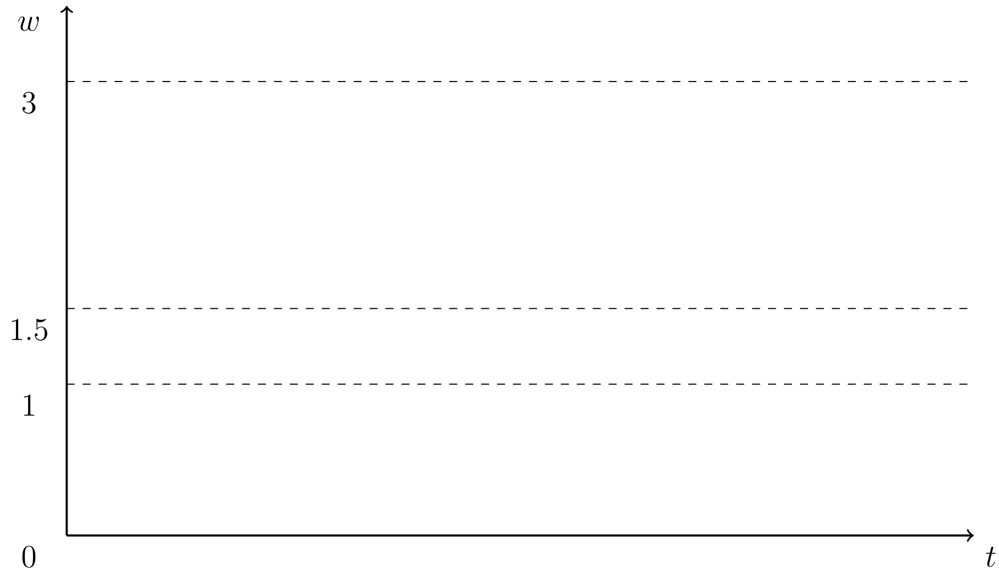
   - If indifferent between signing and not signing a contract, the worker will sign,
   - If indifferent between two types of contract, the worker will choose the contract with the lower task level.

   (c) On the graph below, draw an indifference curve for a type-a worker for each level of utility $u_a = 1.5, 2$. On the same graph, draw an indifference curve for a type-b worker for each level of utility $u_b = 1, 1.5$. Label the vertical intercepts.

   [4 marks]

   (d) What will be the menu of contracts $[(w_a, t_a)), (w_b, t_b)]$ offered by the firms in a separating equilibrium? Represent this equilibrium on the graph below. Compare type-a and type-b workers' payoffs with the no-signalling outcome in part (a).

   [6 marks]



<u>**Section B**</u>

1. **[25 marks]** For the following question consider a consumer making decisions under risk.

   (a) First consider a risk neutral individual with wealth w. Show that this individual's Arrow-Pratt measure of absolute risk aversion, $R_a(w)$, is equal to zero. (Hint: What type of utility function must the individual have in order to be risk neutral?)

   [6 marks]

   (b) Show that for $\beta > 0$, the utility function $u(w) = \alpha + \beta + \beta \log(w)$ displays decreasing absolute risk aversion.

   [6 marks]

   (c) A consumer has a utility function given by $u(w) = \log(w)$ and initial wealth equal to w. This consumer is offered the opportunity to bet on the flip of a coin that has a probability π of coming up heads. If he/she bets an amount of money x, he/she will have $w + x$ if head comes up and $w - x$ if tails comes up. Solve for the optimal amount x that the consumer would be willing to bet, as a function of π.

   [8 marks]

   (d) What is the certainty equivalent of the consumer in the previous sub-question (in terms of π)?

   [5 marks]

2. **[25 marks]** Let an agent consuming in two periods with c_1, c_2 the consumption at each period. The consumer has initial wealth w_1 and can invest this wealth in two assets, a *safe* with certain return R_0 and a *risky* which pays a random return of $\tilde{R}_1$. Suppose that the consumer decides to consume c_1 in the first period and to invest a fraction x of the remaining wealth in the risky asset and a fraction $1 - x$ in the safe. Assume $\delta < 1$ the discount factor. Finally, assume that the preferences of this consumer are represented by the utility function $u(c) = \log(c)$.

(a) Provide an expression for the second-period wealth. (5 marks)

[6 marks]

(b) Set up the intertemporal maximisation problem.

[6 marks]

(c) Derive the first order conditions and provide an interpretation of the optimisation conditions.

[8 marks]

(d) In the game below, reduce the game to its normal form and consider the following mixed strategy of player 1: $\pi(Ll) = \pi(Lr) = \frac{1}{3}, \pi(Rl) = 1/12, \pi(Rr) = 1/4$, where $\pi(.)$ corresponds to the probability that player 1 is playing the respective strategy. Find the corresponding behavioural strategy of player 1.

[5 marks]