

Instructions: Add the last three digits of your student ID. Divide the result by 4 to obtain a remainder  $R$ . The *Type* of exam that you need to solve is equal to  $R + 1$ .

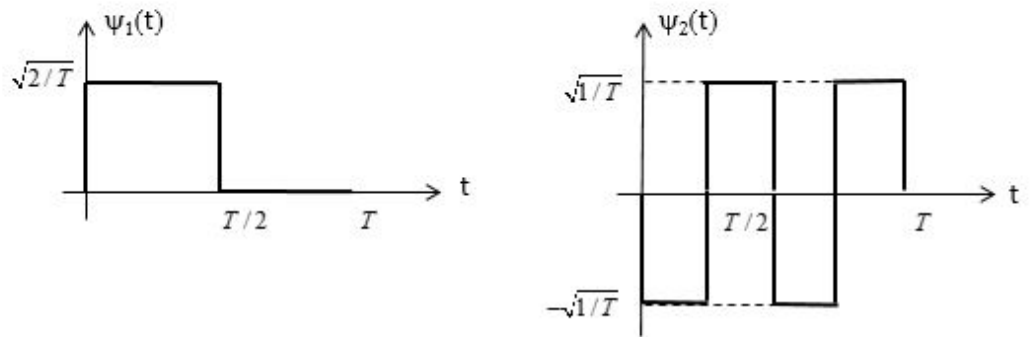
The exam problem statements can be found in the pages below.

Problem # 1	Problem # 2	Problem # 3	Total points
/40	/25	/35	/100

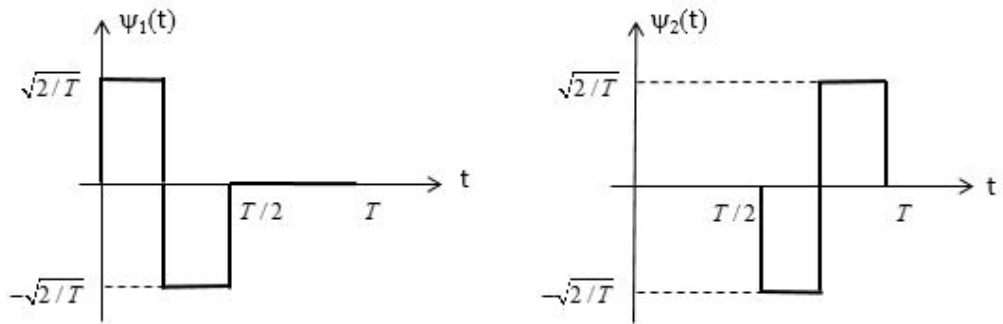
1. Consider a digital communication system using the 16-QAM IEEE 802.11 mapping that was shown in class<sup>1</sup>.

(a) Express the last 4 digits of your student ID in Binary Coded Decimal (BCD) format to obtain a bit string of 16 bits. Sketch the associated waveform  $s(t)$  for the pulses below. For the purpose of your sketch, you may assume that  $T = 1$  and  $E_s = 5$ . (Hint: 4 symbols.  $s(t) = s_1\psi_1(t) + s_2\psi_2(t)$ .)

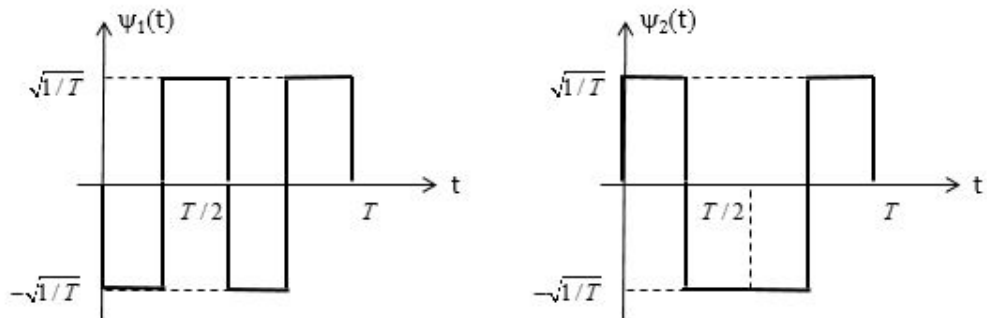
i. Exam type 1:



ii. Exam type 2:

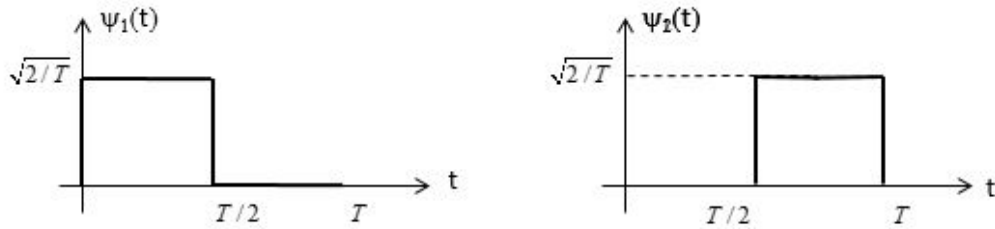


iii. Exam type 3:



<sup>1</sup>See also lecture note 10\_Modulations\_Wireless\_Standards.pdf in Canvas under Files/Lectures.

iv. Exam type 4:



- (b) Evaluate the average bit error probability  $P_b$  for the average signal energy-to-noise ratio value below:
- Exam type 1:  $E_s/N_0 = 20$  dB
  - Exam type 2:  $E_s/N_0 = 15$  dB
  - Exam type 3:  $E_s/N_0 = 12$  dB
  - Exam type 4:  $E_s/N_0 = 17$  dB

For the purpose of your computation, you may use the simple approximation formulas in the Appendix at the end of the exam. Express your answer in terms of the Gaussian  $Q$ -function and evaluate it using either Table 1 in the Appendix or MATLAB.

- (c) With the values from part (a), estimate the most likely information bits sent if the matched filters (or correlators) outputs  $(Y_1, Y_2)$  are equal to
- $(0.5, -1.5)$
  - $(-3.5, -0.5)$
  - $(2.5, 2.5)$
  - $(-2.5, 0.5)$

2. (Pulse shaping) Consider a binary communication link transmitting at 1 Mbps.

- (a) Sketch very carefully the power spectral density of the pulse shaping technique below.
- Exam type 1: Polar RZ
  - Exam type 2: Unipolar NRZ
  - Exam type 3: Polar Manchester
  - Exam type 4: AMI NRZ

For the purpose of your sketch, use a normalized pulse amplitude so that  $a^2T_b = 1$ .

- (b) Add the last four digits of your student ID number. Then divide the result by 10 to obtain a remainder  $R$ . and use a binary representation of  $R$  to obtain a 4-bit sequence. Use the technique in part (a) to sketch the associated waveform.

3. Energy, symbol rate and bandwidth of quadrature amplitude modulation (QAM)

(a) Compute the minimum average signal energy-to-noise ratio  $(E_s/N_0)_{\min}$  in dB that required in order to achieve an average bit error probability  $P_b \leq 2 \times 10^{-4}$  if the mapping is

- i. Exam type 1: 256-QAM
- ii. Exam type 2: 1024-QAM
- iii. Exam type 3: 64-QAM
- iv. Exam type 4: 4096-QAM

For convenience, you may use Table 2 and the simple approximation formulas in the Appendix at the end of the exam.

(b) The bit rate of a communication link using the mapping in part (a) is 40 Mbps. Determine the symbol rate  $R_s$  in symbols per second (baud).

(c) Sketch carefully the overall lowpass channel response  $|X(f)|$  if square-root raised-cosine (SRRC) pulses are used with the rolloff factor  $\alpha$  value below

- i. Exam type 1:  $\alpha = 0.85$
- ii. Exam type 2:  $\alpha = 0.45$
- iii. Exam type 3:  $\alpha = 0.15$
- iv. Exam type 4:  $\alpha = 0.65$

## APPENDIX

**Table 1: Selected values of the Gaussian  $Q$ -function**

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	5.00e-01	4.60e-01	4.21e-01	3.82e-01	3.45e-01	3.09e-01	2.74e-01	2.42e-01	2.12e-01	1.84e-01
1.0	1.59e-01	1.36e-01	1.15e-01	9.68e-02	8.08e-02	6.68e-02	5.48e-02	4.46e-02	3.59e-02	2.87e-02
2.0	2.27e-02	1.79e-02	1.39e-02	1.07e-02	8.20e-03	6.21e-03	4.66e-03	3.47e-03	2.56e-03	1.87e-03
3.0	1.35e-03	9.68e-04	6.87e-04	4.83e-04	3.37e-04	2.33e-04	1.59e-04	1.08e-04	7.23e-05	4.81e-05
4.0	3.17e-05	2.07e-05	1.33e-05	8.54e-06	5.41e-06	3.40e-06	2.11e-06	1.30e-06	7.93e-07	4.79e-07
5.0	2.87e-07	1.70e-07	9.96e-08	5.79e-08	3.33e-08	1.90e-08	1.07e-08	5.99e-09	3.32e-09	1.82e-09

Example:  $Q(2.5) = 6.21\text{e-}03 = 6.21 \times 10^{-3}$ .

**Table 2: Selected values of the inverse Gaussian  $Q$ -function**

$Q(x)$	$x$
$10^{-1}$	1.28
$10^{-2}$	2.33
$10^{-3}$	3.10
$10^{-4}$	3.73
$10^{-5}$	4.27
$10^{-6}$	4.76
$10^{-7}$	5.20
$10^{-8}$	5.61
$10^{-9}$	6.00
$10^{-10}$	6.63
$10^{-11}$	6.71
$10^{-12}$	7.03
$10^{-13}$	7.35
$10^{-14}$	7.65

Example:  $Q^{-1}(10^{-4}) = 3.73$ .

### Average bit error probability approximation formulas

$M$ -PAM:

$$P_b = Q\left(\sqrt{\frac{6}{(M^2 - 1)} \frac{E_s}{N_0}}\right).$$

$M$ -PSK:

$$P_b = Q\left(\sqrt{\frac{2E_s}{N_0} \sin^2\left(\frac{\pi}{M}\right)}\right).$$

$M$ -QAM:

$$P_b = 2Q\left(\sqrt{\frac{3}{(M - 1)} \frac{E_s}{N_0}}\right).$$