Portfolio Analysis

Context

You work in the risk management department of the defined-benefit pension fund DBPF. It is your responsibility to report on the funding position of DBPF to your line manager every working day by 11 o'clock. Your daily routine is as follows: every morning at 9 o'clock, you receive values from DBPF's research department for the parameters of the *Nelson-Siegel model* of the term structure of interest rates:

$$r(\tau) = a + b \cdot \left[\frac{1 - \exp\left\{-\frac{\tau}{\theta}\right\}}{\frac{\tau}{\theta}} \right] + c \cdot \exp\left\{-\tau/\theta\right\}$$

You use then this term structure model to value DBPF's assets. The assets are exclusively bonds. You also use the model to value DBPF's obligations towards its policyholders. Once you have the value of assets and the value of obligations, you compute DBPF's current funding position. Furthermore, you assess the sensitivity of this position to changes in interest rates using the modified duration.

Today's values for the parameters of the *Nelson-Siegel model* are:

a	b	c	θ
0.00024	−0.00420	−0.00190	1.00000

You also expect an intern in the morning who should ask questions about your work. The intern arrives late for work, however, and you have already computed the value of DBPF's obligations to policyholders, which is £850m. You are also nearly finished with the valuation of DBPF's bonds. Only the following three default-free bonds have not been valued yet:

- Bond 1: a zero bond that pays £1000 in 45 days
- Bond 2: a coupon bond that pays a coupon of £50 in 180 days and the same coupon plus the principal of £1000 in 360 days
- Bond 3: a zero bond that pays £1000 in 635 days

As the deadline for the report is approaching fast, you cannot consider the questions of the intern. You value the remaining bonds and find that DBPF's assets have also a value of £850m. You compute that a small upward

shift of today's term structure would imply an approximate modified duration of 4.6 for DBPF's bond portfolio, but of 10.5 for DBPF's obligations to policyholders. You finish your report just in time.

Questions of the intern

The intern did not grasp much of what you do and why you do it. After your lunchbreak, you find a note on your desk with the following text:

``First of all, I apologise again that I was late, but I could not find my Oyster card in the morning and had to ride my bicycle to work. I know that you would have explained it to me, but could you please explain:

1. What is the exact business of DBPF?
2. What is the funding position and why is it important?
3. While I understand what bonds are and why their value depends on interest rates, I do not understand what DBPF's obligations towards their policyholders are.
4. I could not follow the computations of the values for the three bonds. Could you please explain it to me and give me the correct values?
5. I also have no clue about the modified duration. What is it and why is it useful for portfolio risk management of the DBPF? In particular, are the modified durations of 4.6 for DBPF's bond portfolio and of 10.5 for DBPF's obligations to policyholders of concern and why?
6. Finally, what can be done if the durations are of concern?

I would be very grateful if you could answer my questions. I am at work tomorrow and could pick up your response then.''

Your task

You are a friendly person and decide to write a short paper for the intern that answers her questions. The paper should not answer the questions mechanically one by one, but rather present a careful explanation that answers all the questions and places the answers in context.

Format

There is no need for a cover page. The document must be typed with a word processing programme (MSWord, Pages, LaTeX). You must not use your name within the submitted document. Include all tables and charts in the main body of your paper. The word limit is 1,500 words.