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Problem Set #3

1. (20 pts) Suppose that the total demand for products in Maryland is given by the equation $Q = 450 - 5P$, where Q is in millions. Suppose that the total supply function for products in Maryland is $Q = 4P$. For all answers in this question, round to two decimal places.
    a. Suppose that there is currently a 10% sales tax on products in Maryland. What are the **equilibrium prices** received by firms and paid by consumers? What are the current **consumer surplus, producer surplus, government revenue, and deadweight loss** associated with this tax? Who bears more of the **tax burden**?
    b. Suppose there is a proposal to enact a 5% sales tax on services in Maryland and also to lower the tax rate on products to 5%. Suppose that the total demand for services in Maryland is given by the equation $Q = 160 - P$, and the corresponding supply function is $Q = 3P$ (Q is in millions here as well). What would the **new prices and quantities in each market** (products and services) be? Would **total government revenue** under this proposal be higher or lower than it currently is as described in part a? <u>Assume no sales tax on services in part a</u>. Would **total deadweight loss** be higher or lower?

2. (20 points) People visit national parks because they want to experience the magnificence of nature in its unaltered primordial state. The problem is that the more people that come for this experience, the less unaltered nature there is. Suppose that there are N visitors to Yosemite National Park per day. Suppose also that the average consumer surplus (or utility) of a visit to the park per visitor in $ is given by U(N,P) = 200 – (N/200) – P, where P is the price of admission. Assume that people will choose to visit the park as long as they have non-negative utility for the visit. Suppose also that the marginal cost (to the park) per visitor is 0, and that the park has no fixed costs.
    a. What is the demand function for visits to the park?
    b. Since U(N,P) is the average utility (or average consumer surplus) per visitor, when there are N visitors, what is the equation for U_S(N,P), the **total value** of visitor (consumer) surplus **for N visitors** to the park in a given day with the price of admission to the park equal to P?
    c. In terms of P, what is the value of N that **maximizes total visitor surplus**? This will be an expression similar to N = A + (or –) bP, with actual numbers in place of A and b. How many visitors per day would maximize that surplus if admission were free (P=0)? How many people would actually visit the park (use the demand you calculated from part a) per day if admission were free?
    d. If the park service's goal with the park were to **maximize profit**, what admission price (call it p*) would they choose? How many daily visitors (call it N*) would there be? Using your answer from part b, what would total visitor surplus (U_S(N,P)) be with N* visitors at price p*? Using your answer from part c, what would be the total number of visitors (call it N') that would maximize total visitor surplus at price p*? Why is there a difference between N* and N'?
    e. Based on your answers to this problem, how do you think the park service should decide on the price of admission to national parks? Should they choose (or include) another method to manage park attendance?

3. (30 pts) Jiao is bidding at an art auction against one other bidder. She values the painting up for bid at $12 million. No one else at the auction knows her valuation, and she does not know anyone else's valuation. She and the other bidder know that individual valuations for the painting among all art lovers range from $8 million to $18 million, and each value in this range is equally likely to be Jiao's true valuation or her opponent's true valuation. This means that if a bidder has a valuation of V somewhere between $8 million and $18 million, the probability that the other bidder has a higher valuation is ($18 million – V)/$10 million.

   If Jiao wins the painting at a price of p, her utility will be 12 million – p. If she does not win the painting, her utility will be 0.

   a. Suppose this is a first-price sealed bid auction. This means each bidder **submits one bid in writing, without knowing their opponent's bid**, and the bidder who submits the highest bid wins the item and pays the highest bid. What is Jiao's expected utility if she bids $12 million?
   b. True or False: Overbidding (that is, bidding **more than one's valuation** of the painting) <u>is a dominated strategy</u> in this auction format. Explain why you answered the way you did.
   c. True or False: Being truthful (that is, bidding one's exact valuation of the painting) <u>is not a dominant strategy</u> in this auction format. (Hint: Compare bidding $1 less than your valuation to bidding your valuation). Explain why you answered the way you did.
   d. Now suppose that this is an English auction, the style that most of us think of when we think of an auction. The auctioneer calls out a price until somebody accepts the price he calls, at which point he raises the price by a very small amount. He then asks for acceptance of the new price, and when that is accepted, he raises the price again. This continues until no one is willing to accept a new price, at which point the painting is sold to the bidder who accepted the most recent accepted price, which will be the price actually paid by that bidder. Suppose the other bidder has accepted the most recent price the auctioneer called out, and the auctioneer has now called out a slightly higher price. What is Jiao's best response?
   e. True or False: The seller of the painting <u>should NOT expect</u> to receive the full valuation of the bidder who has the highest valuation under either of these auction formats. Explain why you answered the way you did.
   f. In light of your answers in this section, what do you think are the benefits of choosing an auction to sell products? How is an auction like price discrimination? What kinds of products would be good to sell via auction?

4. (30 pts) Suppose two players, Player A and Player B, are playing a game called, "Split the Pot." It works like this: in front of them is a pot with $1500. Player A proposes a division of the pot in the form of (π_A, π_B), where π_A is the amount Player A gets to keep, π_B is the amount Player B gets to keep, and $\pi_A + \pi_B = 1500$. Player B then decides to accept or reject Player A's proposal. If Player B accepts the proposed division of the pot, then Player A gets π_A, and Player B gets π_B. If Player B rejects the proposed division of the pot, then they both get 0.

   Suppose that Player A's utility from an amount of money is equal to the amount of money: U(m) = m. Suppose the same is true for Player B. This means that if Player B accepts Player A's proposal, Player A's utility will be $U(\pi_A) = \pi_A$, and Player B's utility will be $U(\pi_B) = \pi_B$. Suppose for now that there is no other source of utility in this game for either player. **Each player <u>only</u> <u>cares</u> about how much money he or she gets and absolutely nothing else.** Also suppose that if Player B is indifferent between accepting a proposal and rejecting it – that is, **if Player B's utility from accepting a proposal is equal to her utility from rejecting it – then she will accept it**.

   a. Draw the game tree for this sequential game. What will Player B's utility be in Nash Equilibrium?
   b. Now suppose that Player B **has a different utility function** from what was described earlier. She has two components to her utility: **money and comparison**. Let $d = \pi_A - \pi_B$ be the difference between the two players' payoffs in the game. Then assume Player B's utility function is U(m,d) = m – d. The game still functions exactly as it did before, with Player A proposing a split of the pot and Player B deciding to accept the proposal or reject it. Player B still gets 0 utility if she rejects Player A's proposal, and she still accepts a proposal if she is indifferent between accepting and rejecting. Draw a new game tree reflecting Player B's different utility function. What rule will Player B follow in deciding whether to accept an offer? Knowing this, how much will Player A offer Player B in Nash equilibrium?
   c. Now suppose that there are two types of people: Type I, those whose utility functions are U(m) = m, and Type II, those whose utility functions are U(m,d) = m – d. Suppose everyone knows that half the population is the first type, and half is the second type. Suppose Player A knows that he is Type I, but he does not know Player B's type. What is Player A's expected utility (that is, the expected value of his utility) in this case from making his proposal as if Player B were Type I and didn't care about comparison? What is his expected utility from making his proposal as if Player B were Type II and did care about comparison? Which proposal of those two should he make?
   d. How much would Player A be willing to spend on a screening system that would allow him to know Player B's type before he made the proposal?

e. What insights does this model provide about the pitfalls of designing compensation schemes inside your firm?