

ECON2200(2021): Assn3 Production Functions, Factors and Cost Minimization

Due: Monday, March 22nd, 11:59 pm. Please submit to the dropbox at Assessments >> Assignments.

Instructions:

For full marks:

- 1) Please submit your assignment to the drop box under **Assessment >> Assignments**. Do it on paper and then photograph or scan it. Submit as **ONE PDF file**.
- 2) For algebra and other calculations, use **short sentences to guide the reader** (SSTGR) through your logic. Be sure to **state efficiency and equilibrium conditions in words** prior to writing down the mathematical equations. Use sentences like "Substitute" [1] in [2] to find P as follows (where [1] & [2] are equation labels). Provide a **concluding statement with correct units**.
- 3) Graphs should have meaningful titles, axes and curve labels, correct units on axes, and should be plotted to scale unless you are asked for a "sketch". All graphs including sketches should be tidy. You may use the attached graph paper, your own graph paper, or plot to scale on regular paper using a ruler.
- 4) Assignments should be **neat and tidy with questions in order** to help the TAs, [Tapis, Thiago](#) and [Zihao Sheng](#), to mark and to make it easier for review later.
- 5) Be sure to submit your assignments so that the top of all pages is aligned with the top of the computer screen to help the TAs grade. It takes longer to grade online so please make all efforts to make it easy to grade. Thanks!
Your assignment will be marked based on selected questions and for overall completion.
* If you have a question, please post to Discussions and or visit us during our office hours during which we will be happy to help you out. Questions with short answers may be addressed by email.

Problem 1: Isoquant, Isocost Cost Minimizing Approach to Factor Selection:

Suppose Mr Once-ler has a thneeds company called "Thneed Inc.". Thneeds are soft fluffy things of questionable utility that people think they need. They are made from the soft tufts of the truffula trees. Suppose he has received an order to make **10,000 thneeds**. Let Q represent the quantity of these **thneeds** produced per month.

Suppose there are **two factor inputs**, L (labour) and truffula trees (K) where in this case, K is referred to as "**natural capital**". Mr Once-ler needs to decide how human hours of labour (L) [in hours/month] and truffala trees (K) [in trees/month] to hire or cut down respectively to make 10,000 thneeds over a 1 month period.

Let the production function be:

$$Q = f(K, L) = 100 * L^{1/4} * K^{3/4}. \quad [1]$$

A is the **Technical Factor Productivity term** (TFP) and is equal to 100.

The **marginal product of labour** and **capital** are respectively:

$$MP_L(K, L) = 25 * \left(\frac{K}{L}\right)^{3/4} \quad [2]$$

and

$$MP_K(K, L) = 75 * \left(\frac{L}{K}\right)^{1/4} \quad [3]$$

- a) Find the **equation** of the **isoquant** for the production of Q = 10,000 thneeds. Rearrange to get K as a function of L.
- b) **Plot** and label this isoquant on graph paper labeled as **Fig. 1**.
- c) Find the **marginal rate of technical substitution** ($MRTS_{L,K}$) as a function of L and K. This can be thought of as the marginal benefit of hiring an additional labour hour in terms of saved truffala trees while keeping output constant.
- d) Suppose that the hourly rate of labour is $w = \$20/\text{hour}$ and the **opportunity cost** of cutting down a truffala tree is $r = \$40/\text{tree}$. Write down a general equation of an **isocost** where TC represents total costs per hour due to these factor inputs. Fill in values of the K and L intercepts for a variety of total costs shown in Table 2 and then **plot and label** them on Fig. 1 along with your isoquant.

- e) The **marginal opportunity cost of hiring a labour** in terms of truffala trees equals the negative of the slope of the isocost. Find this MOC of hiring labour.
- f) Find the **tangency condition**.
- g) Use the tangency condition and the equation of the isoquant to find the **cost minimizing quantities** of labour and truffala trees and call these L^{LR*} and K^{LR*} where “LR” refers to “long run”.
- h) Find the minimum total costs of producing 10,000 thneeds over a 1 month period. **Plot and label** the long run cost minimizing Isocost on figure 1.
- i) Suppose that in the short run, you can only hire 50 labour hours per month. How many truffala trees will you need to cut down to meet the thneeds order. Calculate your total short run costs. Plot the corresponding isocost on your graph and label the short run cost minimizing bundle.
- j) Does this production function exhibit constant, increasing or decreasing **returns to scale**? Explain briefly.
- k) Are the **marginal products** of labour and truffala trees respectively decreasing, diminishing, or increasing? Explain briefly.

Problem 2:

Suppose that there are two inputs to the final output of "butter product" (Q), pure butter (x) and palm-oil-butter-substitute (y). For simplicity, ignore other inputs. Suppose that the quality of the butter product is independent of the proportion of x and y used. The units for the inputs are kg butter and kg of palm oil-butter-substitute (pbs) respectively. The price of the butter input is $P_x = \$6/\text{butter unit}$ and the price of the palm-oil substitute is $P_y = \$2/\text{pbo unit}$. The production function is: $Q = f(x,y) = x + y$.

- a) Suppose this butter product company wants to make 100 butter product units per second. Find an equation of the corresponding isoquant and plot on a graph.
- b) Plot a family of isocosts on your graph including the isocost that minimizes the costs of producing 100 butter product units per second.
- c) Find the cost minimizing bundle of inputs of pure butter and palm oil substitute and the total cost of producing these.
- d) Based on the article on Buttergate, which country has been invaded by butter product in which the quantity of butter is less than 100%. <https://www.aljazeera.com/news/2021/3/4/buttergate-why-isnt-butter-softening-in-canada>
- e) Do you think that the assumption that **the product quality** should be held constant a good one? Relate your answer to the short video of the Kate and Leopold Butter Scene at: <https://www.youtube.com/watch?v=Fwi0bsJ5F8I>
- f) Palm oil is responsible for large amounts of rainforest deforestation including orangutan habitat. Suppose that high taxes on palm oil are levied to reduce this deforestation and these drive the price of palm oil butter substitute up to $\$8/\text{pbo unit}$. What happens to the slopes of the isocosts. Label the new solution on the graph. Which product is not consumed at all and which is consumed in a positive amount. No calculation is required here.

Good luck!