FINANCIAL INTERMEDIATION

ASSIGNMENT 2

DUE FRIDAY MARCH 19, 12:00 (Noon), 2021

REMEMBER TO HAND IN **USING DIGITAL EXAM**

General Instructions

Please form groups of 2 to 4 students and produce one **pdf** document per group. You should be able to form groups using Digital Exam. Print your **full name and e-mail address** on the front page. I.e., the front page should contain the name of all group members. Hand-in and registration of pass/no pass grades will be administered through **Digital Exam**. This is the first of 4 assignments. You are required to pass at least 2 assignments before you can take the final exam. The assignment will receive a pass/no pass grade (which does not count in the final grade). You will not be allowed to hand in the same assignment again if you did not receive a passing grade. Insights and techniques gained from solving the assignment will be an integral part of the readings for the final exam. Thus, making a strong effort with the problem set will help you in the final exam.

Make sure you answer the questions clearly. Do not just show numbers and tables, or copy formulas from EXCEL and expect a reader to guess the context. State as concisely as possible how you reached the result so that the reader can see your reasoning. The exercise is not just about finding the right answers – it is also aimed at sharpening your ability to present arguments and results clearly.

The document must be completely self-contained, i.e., supporting graphs and tables etc. should be integrated into a single pdf file.

Enjoy!

Problem 1

We consider in this problem a mixed binomial setting for modeling the losses of a loan portfolio. We are interested in comparing two different mixed binomial models. In model 1, the random default probability of each loan is given as a random variable $\tilde{p}_1$ which can take values $0, 0.01, 0.02, \ldots, 0.10$ with probabilities given as

$$P(\tilde{p}_1 = 0.01i) = \begin{pmatrix} 10 \\ i \end{pmatrix} 0.6^i \cdot 0.4^{10-i} \text{ for } i = 0, 1, \ldots, 10.$$

Another way of expressing $\tilde{p}_1$'s distribution is to say that it is equal to that of $\frac{1}{100}\tilde{B}_1$ where $\tilde{B}_1$ is a binomial distribution with $N = 10$ and $p = 0.6$. Similarly, in model 2 we assume that the random default probability $\tilde{p}_2$ is given as another scaled binomial distribution:

$$P(\tilde{p}_2 = 0.01i) = \begin{pmatrix} 20 \\ i \end{pmatrix} 0.3^i \cdot 0.7^{20-i} \text{ for } i = 0, 1, \ldots, 20,$$

which we recognize as being equal to that of $\frac{1}{100}\tilde{B}_2$ where $\tilde{B}_2$ is a binomial distribution with $N = 20$ and $p = 0.3$.

1. Compute the mean and variance of $\tilde{p}_1$ and $\tilde{p}_2$.

2. In a portfolio of 50 loans, compute the expected number of defaults and the variance of the number of defaults under both models.

Now assume that the loan portfolio considered is very large, i.e., consists of a very large number of small loans of equal size, such that both in model 1 and in model 2 the loss fraction ($\frac{D_N}{N}$ in the notation of the notes) is well approximated with the distribution of $\tilde{p}_1$ and $\tilde{p}_2$, respectively.

3. Under the two models, what is the probability that the fraction of loans that default is smaller than 1.5%?

4. Under the two models, what is the probability that a fraction larger than 9.5% defaults?

5. Assume that we construct a CDO with the portfolio of loans as collateral, and we design a senior tranche which repays in full as long as the loss fraction is below or equal to 7%. but which takes losses thereafter. Which model would you expect to assign the highest expected pay-off to the senior tranche? Explain your answer.

Problem 2

In this problem you will use simulation to compute expected pay-offs to tranches in a Vasicek model using the large homogeneous portfolio (LHP) approximation. Recall that given parameters $\bar{p}$ and ρ, the LHP approximation means that we can simulate the fraction of loan losses in our (large homogeneous) portfolio by simulating the outcome of a standard normal random variable M and computing for each outcome m the expression

$$p(m) \quad = \quad \Phi\Big(\frac{\Phi^{-1}(\bar{p}) - \sqrt{\rho}m}{\sqrt{1-\rho}}\Big).$$

If we want to incorporate a recovery rate R on each loan, then the fractional loss in the loan portfolio corresponding to the outcome m is just $(1-R)\Phi\big(\frac{\Phi^{-1}(\bar{p})-\sqrt{\rho}m}{\sqrt{1-\rho}}\big)$. We will assume throughout that $R = 0.4$.

We consider a large portfolio with a total principal of 1 (which you can think of as 1 billion, if that makes it easier to imagine it as composed of many small loans). Consider a senior tranche with a principal of 0.9, a junior tranche with a principal of 0.01 and an equity tranche with a principal of 0.09. Assume to begin with that $\bar{p} = 0.1$ and $\rho = 0.2$.

1. Consider three outcomes of $m = -1.9, m = -1.0, m = 0.5$ of M. Compute for each outcome the realized pay-off on the entire loan portfolio, and on each of the three tranches. Present the result in a single table.

2. Using at least 1000 simulations (say how many you use), compute (by averaging over the pay-offs in each simulated outcome) the expected pay-offs to each of the three tranches.

3. Report also the simulated expected pay-off to the entire portfolio. Why does this give you a hint about your simulation accuracy?

4. Show in one graph the expected pay-offs to each tranche scaled by the principal (i.e., expected pay-off divided by the principal) as a function of

the correlation ρ. Let the correlation vary between 0.05 and 0.75. (Hints: If you use EXCEL and have not used Datatable before, this is time to do it! There are many introductions on youtube. Also, it may give a nicer graph if you use the same simulation outcome of M when computing expected values for different values of the correlation. One way is to simulate the 1000 or more outcomes of M once and then copy and paste as values to a different column, that you then use for all your computations.)

5. Comment on the graph.

6. Voluntary extra question: Compute for each tranche (using the simulations) the standard deviation of the (gross) return, i.e., the realized pay-off divided by the expected value at time 0.