Formula Reference Sheet

- **Naïve Forecast:** $\hat{Y}_t = Y_{t-1}$
- **Exponential Smoothing Forecast:** $\hat{Y}_t = \alpha Y_{t-1} + (1-\alpha)\hat{Y}_{t-1}$
- **Residual:** $e_i = (y_i - \hat{y}_i)$
- **Mean Forecast Error:** $MFE = \frac{\sum_{i=1}^{n} e_i}{n}$
- **Mean Absolute Error:** $MAE = \frac{\sum_{i=1}^{n} |e_i|}{n}$
- **Mean Squared Error:** $MSE = \frac{\sum_{i=1}^{n} (e_i{}^2)}{n}$
- **Root Mean Squared Error:** $RMSE = \sqrt{\frac{\sum_{i=1}^{n} (e_i{}^2)}{n}}$
- **Simple Regression:** $\hat{y}_i = \beta_0 + \beta_1 x_i$
- **Multiple Regression:** $\hat{y}_i = \beta_0 + \beta_1 x_{1i} + \cdots + \beta_k x_{ki}$
- **Sum of Squares Total:** $SST = \sum_{i=1}^{n} (y_i - \bar{y})^2$
- **Sum of Squared Errors** $SSE = \sum_{i=1}^{n} (e_i{}^2)$
- **R-Squared:** $R^2 = \left(1 - \frac{SSE}{SST}\right)$
- **Economic Order Quantity:** $Q^* = \sqrt{\frac{2DC_O}{C_H}}$
- **Economic Production Lot Size Quantity:** $Q^* = \sqrt{\frac{2DC_O}{\left(1-\frac{d}{p}\right)C_H}}$
- **Annual Cost of Inventory (including COGS):** $TC = \left(\frac{1}{2} Q * C_H\right) + \left(\frac{D}{Q} * C_O\right) + \left(\frac{Cost}{Unit} * D\right)$
- **Optimal Quantity via Incremental Analysis:** $Q^* = Q \text{ such that } [P(demand \leq Q)] = \frac{C_U}{C_O + C_U}$
- **Net Profit:** $Profit = (Price * Demand) - [(Var.\,cost\ per\ unit * Demand) + Total\ Fixed\ Costs\ ]$

Question 1: For parts 1.1–20, fill in the blank with the letter of the term that best matches the concept described. (1 pt each, 20 pts total)

____**1.1.** Describes a model or system that incorporates randomness

____**1.2.** A practical limitation placed on an available resource

____**1.3.** Describes a rule-based model or system that does not incorporate randomness

____**1.4.** Any method that aims to reflect underlying movement patterns while minimizing random noise

____**1.5.** The set of decision variable values that simultaneously satisfy all practical limitations imposed on a model

____**1.6.** A classification given to time series displaying neither seasonality nor trend

____**1.7.** Cyclical trends that occur in repeated (typically annual) patterns

____**1.8.** The expenses associated with storing inventory until it is sold

____**1.9.** The quantity of inventory that is sold between the time of an inventory purchase and its arrival

____**1.10.** A state of having less inventory than is demanded

____**1.11.** A set of observations of a single variable over equally-spaced intervals of time

____**1.12.** The expenses directly associated with producing or manufacturing inventory

____**1.13.** The administrative expenses associated with purchasing inventory

____**1.14.** The increase in objective value when one extra unit of a resource is made available

____**1.15.** The length of time that elapses between inventory orders

____**1.16.** The length of time a supplier requires to deliver inventory after it is ordered

____**1.17.** The distance between the observed value of an observation and a model's predicted value for that observation

____**1.18.** The linear combination of decision variables that a model attempts to either minimize or maximize

____**1.19.** The proportion of orders that a business is able to fulfill

____**1.20.** A state of having more inventory than is demanded

A. Order costs

B. Lead time demand

C. Surplus

D. Seasonality

E. Stationarity

F. Residual

G. Lead time

H. Shortage

I. Feasible Region

J. Cycle time

K. Time Series

L. Constraint

M. Shadow Price

N. Holding costs

O. Service level

P. Stochastic

Q. Smoothing

R. Deterministic

S. Objective Function

T. Cost of goods

Question 2.

The following table displays a record of sales of a certain product over five weeks:

Week	Units Sold
1	6
2	8
3	5
4	9
5	12

Using the naïve forecasting method, generate forecasts for weeks 2-5 and calculate the following three accuracy metrics for your forecasting model:

a. **Mean Forecast Error (MFE):** (2 pts)

b. **Mean Absolute Error (MAE):** (2 pts)

c. **Root Mean Squared Error (RMSE):** (2 pts)

Question 3.

A company purchases a certain raw material for use in manufacturing its own inventory. The cost of this raw material fluctuates over time. The following table displays the average unit cost of the raw material each quarter for the past year:

Quarter	Unit Cost
1	$30
2	$28
3	$29
4	$23

a. **Using exponential smoothing with an alpha value of 0.2, generate the forecasts for periods 1-5.** (3 pts)

Question 4.

Ferris-Steed is a manufacturer of high-quality fabrics. Although the company sells a limited line of finished clothing (e.g., coats, hats), its primary source of revenue is sales of fabric to high-end tailors, suit manufacturers, and luxury fabric stores. Ferris-Steed sells its fabric in bolts, typically 100 yards of woven fabric wrapped around cardboard tube, which can be measured out and cut as needed. Ferris-Steed creates fabric bolts from two sources: a portion of bolts are made using raw materials that have been **manufactured** in-house (through a subsidiary company), and a portion of bolts are made using raw materials that have been **purchased** through an external provider. Each bolt made from manufactured raw materials costs \$10 to produce and contributes \$11 to earnings, while each bolt made from purchased raw materials costs \$20 to produce and contributes only \$8 to earnings. The company has budgeted a weekly maximum of \$2,000 for producing bolts. While each bolt made from purchased materials is more expensive, purchased raw materials have an advantage in that they arrive with most of the processing already complete, and thus take less time to convert to a finished good. Each bolt made from manufactured raw materials takes approximately 90 minutes to produce, whereas each bolt made from purchased raw materials requires only 60 minutes to produce. Total time spent each week manufacturing fabric bolts is limited to 150 hours. The company's customers are fickle, so management considers shortages more 'expensive' (in terms of lost goodwill) than surpluses. As a result, the company sets minimums around its supply plan: management has decided that weekly production must at least 75 bolts.

Management is curious to see what weekly production plan an optimization model would recommend. Specifically, they would like to know how many bolts to produce using manufactured vs. purchased raw materials in order to maximize weekly profit contribution.

a. **Formulate this problem as a linear optimization model, in terms of an objective function and constraint functions.** (5 pts)

b. **Graph the problem using the chart provided (see next page) and identify the feasible region.** (5 pts)

c. **Solve for the optimal number of bolts made from purchased vs. manufactured raw materials the company should plan to produce each week. Provide the optimal number of bolts produced from each source of raw materials, and the corresponding profit contribution achieved through this plan.** (5 pts)

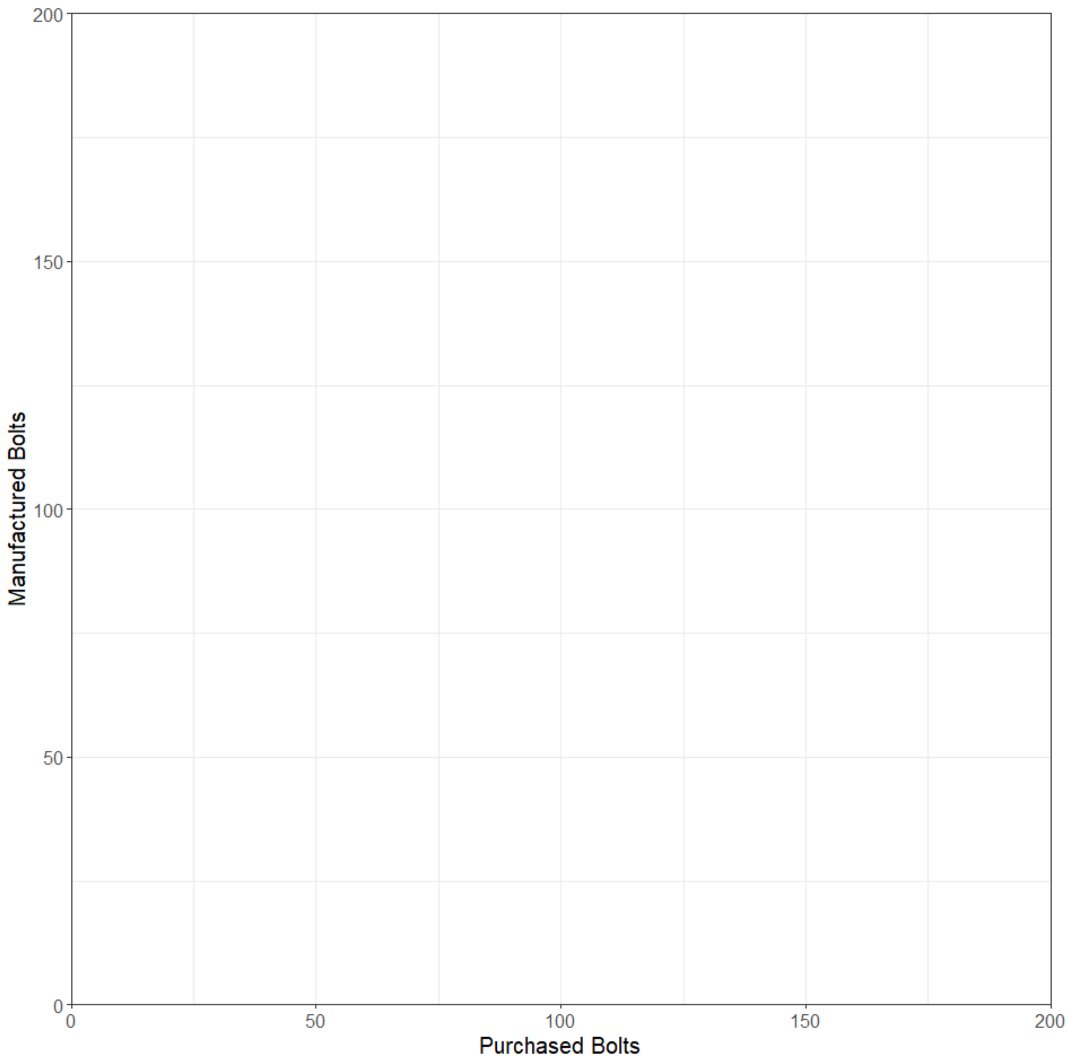
d. **Does this production plan leave slack in any of the constraints incorporated into the model?** (4 pts)

e. **Below is a table of the objective coefficient ranges for this solution. In a few sentences, provide an interpretation of these figures. Imagine you are explaining this to someone who is not familiar with this type of analysis.** (3 pts)

	Lower Bound	Upper Bound
Manufactured Materials	\$4	\$12
Purchased Materials	\$7.33	\$22

f. **The table below displays each constraint's shadow price. In a few sentences, provide an interpretation of these figures (Imagine you are explaining this to someone who is not familiar with linear optimization), and answer how these specific shadow prices should inform management's strategy if they wish to achieve an even higher weekly profit contribution than the maximum possible value indicated by the existing model.** (3 pts)

Constraint	Shadow Price
Supply Minimum	\$0
Capital	\$0.50
Labor Hours	\$0.12


Manufactured Bolts
200
150
100
50
0
0
50
100
150
200
Purchased Bolts

Question 5.

A certain university maintains a counseling center that provides free mental health care to all students enrolled at the university. The center's most popular service is a one-hour counseling session with a trained psychologist. Many students use these sessions on an ongoing, weekly basis, while others make one-time appointments as the need arises. The center's management team is taking an analytic approach to creating a staffing plan, and has created a basic model that forecasts demand for counseling services in the coming year. Management has defined "demand" for services as the average number of one-hour sessions that are scheduled each week over the duration of a quarter. There is a clear seasonal effect on demand that follows the natural flow of an academic year: demand for counseling is always lowest in the Summer quarter (when most students are on break), then steadily increases beginning in the fall quarter, reaching its highest point in the spring quarter, when students generally report feeling the highest levels of stress and burnout. (For reference, the mapping of quarters to academic periods is provided below.) Having accurate forecasts of quarterly demand for counseling will help management maintain a sufficiently sized staff of psychologists to ensure the center can continue providing adequate services to all students in need. The forecast model was built using 10 years (40 periods) of demand data, with the last period representing demand in Winter 2021. The model output is displayed below:

Quarter	Academic Period
Q1	Winter
Q2	Spring
Q3	Summer
Q4	Fall

```
Call:
lm(formula = demand ~ q2 + q3 + q4 + t, data = df)

Residuals:
    Min      1Q  Median      3Q     Max 
-41.333 -23.859  -2.867  19.185  48.086 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 105.7481    11.1687   9.468 3.47e-11 ***
q2           79.1867    12.1784   6.502 1.69e-07 ***
q3          -50.0265    12.1956  -4.102 0.000232 ***
q4          -18.7398    12.2244  -1.533 0.134270    
t             3.1133     0.3746   8.311 8.46e-10 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 27.22 on 35 degrees of freedom
Multiple R-squared:  0.8407,    Adjusted R-squared:  0.8225 
F-statistic: 46.18 on 4 and 35 DF,  p-value: 1.715e-13
```

a. **Express the relationship between quarterly demand for counseling services and both seasonal and trend effects as a linear equation, using the values provided in the model output above.** (3 pts)

b. **Use the model output to generate forecasts for the next four quarters (Spring 2021 to Winter 2022)** (5 pts)

c. **Assuming the model is trustworthy, does demand for counseling appear to be increasing or decreasing over time? Briefly explain how you derive that conclusion from the model output.** (2 pts)

d. **Imagine you are on the management team at the counseling center and are walking through the results of your analysis with a colleague at the university who is not familiar with forecasting, and does not know how to read the table above. Using Spring quarter as your example, briefly (2-3 sentences) provide an interpretation of the coefficient value shown in the model output above.** (2 pts)

Question 6:

Ruby's Craft & Design is a U.S.-based mass producer of Halloween costumes (think pirates, nurses, zombies, and so on.) Each year, the company produces hundreds of thousands of pre-packaged "all in one" costume kits, which are sold to Halloween supply stores around the country, who in turn sell the kits to consumers at a marked-up price. Each year, the company's managers tailor their production plan in an attempt to capitalize on any cultural trends that may result in increased demand for costumes reflecting specific characters or themes. In January 2021, a particularly amusing photograph of Vermont senator Bernie Sanders appearing to look quite unimpressed at the Presidential Inauguration ceremony became an overnight internet sensation and the subject of countless memes. Management immediately speculated that there might be a surge in demand for Bernie Sanders costumes for Halloween 2021. Through a hurried development process involving the company's design and production teams, management derived rough estimates for the costume kit's potential profit.

Management estimates that the cost of materials and labor to produce each Bernie Sanders costume kit would be uniformly distributed between $8 and $12. The sales department estimates they could sell the kits to Halloween stores for approximately $32 per kit. Currently, management is considering a production plan of 20,000 kits, though demand is uncertain. For the sake of simplicity, demand is assumed to be normally distributed with a mean of 18,000 units and a standard deviation of 1,500 units. In the event that management overestimates demand and is left with surplus inventory, the company will sell all remaining units at a discounted price. The discount price would start at $25 and would continue to decrease over time until all leftover kits are sold. As management is unsure how long it would take to deplete the remaining supply of leftover kits, the overall average discount price is therefore uncertain. Representatives from the sales department have compiled a distribution of possible average discount prices for leftover kits, which is displayed in the table below.

Average Discount Price	Probability
$25	0.60
$15	0.25
$10	0.15

Imagine you are creating a simulation model in R. Use the information provided above to answer the following questions. For questions dealing with object creation, ignore the 'assignment operator' portion of each command.

a. **You wish to create 10,000 simulations of the variable cost per kit, which you will store in an object called** `variable_cost`**. Which of the following commands correctly calculates these values? (Ignoring rounding)** (1 pt)

```
a. dunif(n=10000, min=8, max=12)
b. runif(n=10000, min=8, max=12)
c. rnorm(n=10000, mean=8, sd=12)
d. dnorm(x=10000, mean=8, sd=12)
```

b. **You wish to create 10,000 simulations of the quantity of kits demanded, which you will store in an object called** `demand`**. Which of the following commands correctly calculates these values?** (1 pt)

```
a. dnorm(n=10000, mean=18000, sd=1500)
b. pnorm(n=10000, mean=18000, sd=1500)
c. qnorm(n=10000, mean=18000, sd=1500)
d. rnorm(p=10000, mean=18000, sd=1500)
```

c. **You have stored your planned supply value in an object called** `supply`**. Which of the following commands will correctly adjust the values of your** `demand` **object so that no simulated demand value exceeds the planned supply value of 20,000 kits?** (1 pt)

   a. `max(demand, supply)`
   b. `min(demand, supply)`
   c. `pmax(demand, supply)`
   d. `pmin(demand, supply)`

d. **You wish to create 10,000 simulations of the average discount price of leftover kits, which you will store in an object called** `salvage_price`**. Which of the following commands correctly calculates these values?** (1 pt)

   a. `sample(x=c(25, 15, 10), size=10000, prob=c(0.60, 0.25, 0.15), replace=TRUE)`
   b. `sample(x=c(0.60, 0.25, 0.15), size=10000, prob=c(25, 15, 10), replace=TRUE)`
   c. `sample(x=10000, size=c(25, 15, 10), prob=c(0.60, 0.25, 0.15), replace=FALSE)`
   d. `sample(x=c(25, 15, 10), size=c(0.60, 0.25, 0.15), prob=10000, replace=TRUE)`

e. **Using your existing** `supply` **and** `demand` **objects, you wish to create an object called** `salvage_quantity` **that stores the total number of leftover kits in each simulation. Which of the following commands correctly calculates these values?** (1 pt)

   a. `ifelse(supply < demand, (demand-supply), supply))`
   b. `ifelse(supply > demand, supply, demand)`
   c. `ifelse(supply < demand, (supply-demand), (demand-supply))`
   d. `ifelse(supply > demand, (supply-demand), 0)`

f. **You wish to create an object called** `profit` **that calculates the gross profit earned in each simulation (this model ignores fixed costs). Which of the following commands correctly calculates these values?** (1 pt)

   a. `(32*demand) + (salvage_price * salvage_quantity) - (variable_cost * demand)`
   b. `(32*demand) + (salvage_price * salvage_quantity) - (variable_cost * supply)`
   c. `(32*supply) + (salvage_price * supply) - (variable_cost * supply)`
   d. `(32*(supply-demand)) + (salvage_price * salvage_quantity) - (variable_cost)`

g. **Using your** `profit` **object, which of the following commands correctly calculates the probability that the company will actually realize a gross profit?** (1 pt)

   a. `mean(profit)`
   b. `sum(profit) / 10000`
   c. `sum(profit>0) / 10000`
   d. `profit / sum(profit)`

Question 7:

You purchase units of a certain raw material from a supplier at a cost of $4 per unit, which you use to produce your finished goods. On average, you use 50,000 units of this raw material annually to produce your own inventory. Each time you order a new shipment of raw materials, you incur an administrative cost of $800 regardless of the order size. You attribute 20% of the cost of capital to storing idle raw materials.

- **a.** How many units should you purchase in each order in order to minimize your total annual cost of inventory? (5 pts)

- **b.** If you proceed with the order quantity you solved in part (a), how many orders will you need to place each year? (2 pts)

- **c.** If you proceed with the order quantity you solved in part (a), what will your total cost of inventory be each year, including cost of goods? (3 pts)

Question 8:

Below are a series of brief descriptions of scenarios that require (or could benefit from) one of the methods we have discussed this quarter. Map each scenario to one option from the list provided. **You do not need to solve the problems, only match them.** (1 pt each, 12 pts total)

- **a. Decision Tree**
- **b. Binomial Distribution**
- **c. Poisson Distribution**
- **d. Uniform Distribution**
- **e. Exponential Distribution**
- **f. Z-Test (Normal Distribution)**
- **g. Classification Model (e.g., Naïve Bayes)**
- **h. Sensitivity Analysis**
- i. **Forecasting**
- j. **EOQ Model**
- k. **Optimization Model**
- l. **Simulation Model**

___8.1. A fire department is trying to estimate its staffing needs in the peak months. On average, there are 23 fires spread across the peak season (July – October). The department would like to know the probability of being called out to 10 or more fires in the coming month.

___8.2. A meal delivery service offers a wide variety of meal options each week. Many of the meal options share the same raw ingredients, though in different proportions. Management would like to know how much of each meal option it should stock in order to minimize the total cost of raw ingredients, while still satisfying all necessary criteria, including expected demand, nutrition standards, etc.

___8.3. The director of operations for a small business is reorganizing her approach to inventory management. She would like to create a strategy for how many units she should purchase every time she places an order for new inventory in order to minimize her total overhead costs of acquiring raw materials.

___8.4. The loans division of a bank is looking to predict whether entrepreneurs requesting loans are likely to actually pay the loan back, or if they'll default. They want to see if the information they typically have about any given customer can be useful for making such predictions. For example, they typically know a customers' age, marital status, the amount of money in their bank account, their salary, etc.

___8.5. You've built a model that contains a large number of educated assumptions. Your manager is concerned that the results will not be valid if there are significant changes in market level factors in the coming year. In order to support your findings, she has asked you to also include additional recommendations that speak to how much flexibility exists in those assumptions, and what degree of change would be necessary before the model returns a new recommendation.

___8.6. The average weight of a newborn baby (in the US) is normally distributed with a mean of 3,250 grams and a standard deviation of 375 grams. You are interested in estimating the probability of a newborn registering a birth weight of over 4,500 grams.

___8.7. A slot machine manufacturer needs to determine the expected profits and losses associated with the machine's current programming. Slot machines use a random number generator, the output of which maps to a combination of symbols (e.g., 3 diamonds in a row); certain combinations correspond to specific cash payouts, while most map to a payout of $0 (the player gets nothing). The manufacturer is interested in answering questions like 'what's the probability of the machine returning a payout greater than $0?', or 'in what percentage of cases will the machine return a payout greater than $100?', or 'what's the machine's average profit?'

___8.8. You're hosting a charity event, and you've advertised that a certain famous musician will attend and perform at the event. That celebrity is a big draw; as of now, you've sold about $25K in tickets. However, this celebrity is notoriously fickle, and you're worried he may cancel at the last second. If that happens, you guess you'd have to refund about $10K in sales to angry attendees. Just in case, you're debating booking a backup performer… between speaking fees and hotel/travel accommodations, that would also cost you about $5K. But is it worth it to go through that trouble? After all, you think there's only about a 25% chance that the current musician will cancel…

___8.9. An event planner at a hotel has been asked to recommend how many invitations should be issued to try to fill a space with a maximum capacity of 150 seats. She knows that on average, 85% of invitees accept. She is considering issuing 180 invitations, and wants to know the probability of receiving a certain number of acceptances. If she sends 180 invitations, what's the probability that 130 people will accept? 150? All 180?

___8.10. A theme park has aggregated historical data showing the total number of visitors who entered the park each month for the past 6 years. Management is interested in predicting the total number of visitors at the park in each month for the coming 3 months.

___8.11. The manager of a car dealership is taking an analytic approach to planning her staffing needs. She would like to make sure there are enough staff members present on the lot to quickly greet and begin working with potential customers as soon as they arrive on the premises. As part of that analysis, she would like to estimate the typical length of time that elapses between customer arrivals.

___8.12. A web developer works for an online dictionary of medical terms called Sickipedia. She's been asked to create a feature that selects and displays a random entry from the dictionary when the user clicks a button. Each entry is indexed with an integer value (i.e., entry 1-5,000), which the algorithm would reference when selecting which entry to display.

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