MEMO Number CMPE320-S21-0101
DATE: 28 February 2021
TO: CMPE320 Students
FROM: EFC LaBerge
SUBJECT: Histograms, PDFs and PMFs

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1 INTRODUCTION

This project will explore the pdf of functions of a random variable include the pdf of a mixture random variable.

Warning #1: Thinking is required!

Warning #2: Follow directions!

Warning #3: It isn't on the web, so don't bother looking. You may, however, look up background material, such as the definition of terms, etc. All of the terms are well-defined in your textbook and the lectures.

This project involves concepts spread across the various lectures in Module 2, and on the solution to Project 1. It is, however, perfectly acceptable (and desirable) to start early, to do what you can and then go back and do more as the course content expands.

Remember, there are no exams in S21 CMPE320, so I'm looking for you to develop and explain concepts we have developed in class.

This project involves analytical computation as well as simulation. Express your math well! If necessary, you may hand-write your equations and insert them as pictures in your writeup.

> You **may not** collaborate with any other CMPE320 student or students, nor consult with any other humans other than Ehsan and Dr. LaBerge. You may ask all the questions you like in office hours, review sessions, but the *preferred* method of clarification is via the Ask the Professor discussion forum on Blackboard. **Do your own work!**

2 FUNCTIONS OF A RANDOM VARIABLE

The engineers at Universally Marvelous Broadcasting and Communications (UMBC) are designing how to detect the amplitude or the power of a bipolar signal of known amplitude that is corrupted by Additive White Gaussian Noise (AWGN)[1]. Three methods have been suggested:

1) When the signal is received, it is passed the signal through a perfect diode detector, and only the the positive values are used; or,

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[1] The model for a signal with AWGN is
$r(t) = (\pm A) + n(t),$ where $r(t)$ is the received signal, $(\pm A)$ is the desired signal, and $n(t) \sim N(0,\sigma^2)$

2) When the signal is received, the processor computes the amplitude by taking the absolute value of measured signal; or,

3) When the signal is received, the processor computes the amplitude squared by taking the square of the measured signal, thus producing an estimate of the *power*.

The engineers have determined that method 1 will cost $10 in production, but that method 2 will cost $20 in production and method 3 will cost $40 in production. Any of the methods will produce a result that meets the product requirements.

For all of the following questions, assume that the known amplitude is $A = 2\text{V}$, that the known amplitude is equally likely (*hint!)* to be $+A$ or $-A$, and the noise variance is $\sigma^2 = \frac{9}{16}$.

2.1 Method 1

2.1.1 Analytical PDF

Using the CDF method developed in class, analytically derive the probability density function for $s(t), f_S(s),$ where $s(t)$ is the signal that is actually processed using the first method. For this element, please use the symbolic (not numeric) values of A and σ^2.

Expressing the appropriate functional expression of Method 1 as $Y = g(X)$, compute $Y = g(E[X])$, that is the function evaluated at the expected value of the random variable X. Save this value for use in 2.4

2.1.2 Simulated PDF

Using the techniques developed in Project 1, generate a large number of random trials from an appropriate distribution and simulate the probability density function $f_S(s)$. Plot the histogram-based pdf, and then plot the analytical pdf you derived in 2.1.1 on the same set of axes. Provide a professional plot.

Compute the mean of the simulated $s(t)$ from the random trials and save for use in Section 2.4.

2.2 Method 2

2.2.1 Analytical PDF

Using the CDF method developed in class, analytically derive the probability density function for $s(t), f_S(s),$ where $s(t)$ is the signal that is actually processed using the second method. For this element, please use the symbolic (not numeric) values of A and σ^2.

Expressing the appropriate functional expression of Method 2 as $Y = g(X)$, compute $Y = g(E[X])$, that is the function evaluated at the expected value of the random variable X. Save this value for use in 2.4

2.2.2 Simulated PDF

Using the techniques developed in Project 1, generate a large number of random trials from an appropriate distribution and simulate the probability density function $f_S(s)$. Plot the histogram-based pdf, and then plot the analytical pdf you derived in 2.2.1 on the same set of axes. Provide a professional plot.

Compute the mean of the simulated data $s(t)$ from the random trials and save for use in Section 2.4.

2.3 Method 3

2.3.1 Analytical PDF

Using the CDF method developed in class, analytically derive the probability density function for $s(t), f_S(s),$ where $s(t)$ is the signal that is actually processed using the second method. For this element, please use the symbolic (not numeric) values of A and σ^2.

Expressing the appropriate functional expression of Method 3 as $Y = g(X)$, compute $Y = g(E[X])$, that is the function evaluated at the expected value of the random variable X. Save this value for use in 2.4

2.3.2 Simulated PDF

Using the techniques developed in Project 1, generate a large number of random trials from an appropriate distribution and simulate the probability density function $f_S(s)$. Plot the histogram-based pdf, and then plot the analytical pdf you derived in 2.3.1 on the same set of axes. Provide a professional plot.

Compute the mean of the simulated data $s(t)$ from the random trials and save for use in Section 2.4.

2.4 Looking Ahead: Jensen's Inequality

For each of three methods, compare the expected value of the simulated data with the evaluation of the function at the expected value. Is there a consistent inequality relationship that extends across the three cases. Can you guess the general rule, which is known as Jensen's Inequality.

Note: Demonstrations by simulated examples are only examples, never proofs. We will prove Jensen's Inequality in class (or at least I will!).

3 INSTRUCTIONS FOR PROJECT REPORT

3.1 Report Format

The project report shall be in the same form as this document, with an introduction, simulation and discussion section, and a "what I learned" section. Each section shall contain the content identified in Section 2, and described in more detail below subsection below. The report shall be in Times

New Roman 11 point font. MATLAB pictures shall be pasted in-line in the report (this is a useful skill to know!); shall be numbered consecutively; shall be appropriately titled; the axes shall be appropriately labeled; the curves shall be appropriately identified by an appropriate legend. I'll provide suggestions on professional-looking plots on Blackboard. Please follow them.

3.2 Section 2 Content

Section 2 of the report shall be titled "Simulation and Discussion" and shall contain the required derivations, the required simulation plots and a discussion of each plot. The discussion shall address any points identified in Section 2 and any other interesting observations that occur to you. Remember, I know this stuff: you don't. So take a look at the plots and tell me what you see and what it means to you. A large part of the grade is based on what you observe, so take your time!

The subsections of Section 2 of the report should match the subsections of Section 2 of this document.

3.3 Section 3 Content

Section 3 of the report shall be titled "What I learned" and shall contain a summary of what information you observed, what insights you gained, etc. Section 3 shall also contain a subsection critiquing the project and suggesting improvements that I could institute for next spring. Finally, Section 3 shall contain an estimate of how much time you spent on the project, including reading, research, programming, writing, and final preparation.

3.4 Questions

I will accept questions regarding the project via the Ask The Professor Forum on Blackboard, so that I can reply to the entire class, and so that no student has an advantage by clever questioning. I will not do the project for you. I will not be answering (or even acknowledging) individual e-mails, so don't ask.

I will *stop* responding to questions at 12Noon on the day before the project is due. If you don't start early, don't ask for clarification, or read the earlier clarifications.

3.5 Project Grading

The project shall be graded in the following way:

75% of the project score shall depend on the technical, theoretical, and graphical presentations of the tasks set out in Section 2 of this document.

25% of the project score shall be based on an evaluation of the technical writing against the Rubric on Technical Writing, posted on Blackboard, including grammar, clarity, organization, etc. For the purpose of this document, you can assume that the intended audience consists of your CMPE320 classmates.

I will be assisted in grading by the TA/grader staff for this course, but I will personally grade two projects for every student, spread over the five total projects.

3.6 Project Delivery

The project shall be delivered by 11:59 PM on the date indicated in the Detailed Schedule spreadsheet.

Delivery shall be by submission of a PDF file as a Blackboard assignment. This is an individual assignment. You should also publish and deliver your MATLAB files in a single PDF file in the same assignment.

3.7 Academic Integrity

The academic integrity provisions you signed at the beginning of class are in effect. You may discuss the interpretation of the assignment and approaches to solve the various problems amongst yourselves. You MAY NOT share MATLAB code, plots, text, etc. Do your own work.

I *will* be looking at MATLAB files for similarities, so please do not even attempt to copy work.

You may not ask for assistance from the TA/grader staff, although you may ask for help with the various concepts. So, for example, if you don't understand a Gaussian pdf you may ask for help understanding pdfs in general and Gaussian pdfs in particular. You may not ask for help completing the tasks assigned in this document. You *should* ask me for assistance via the Ask The Professor forum. You may ask for help on the concepts during my open office hours.

Do your own work!