Write a report of up to 2000 words on the modelling problem below. In no circumstances should your report exceed 3000 words; you may be penalised by up to 10 marks if it does. If a report obviously exceeds that limit, then your tutor will not mark any material beyond 3000 words. (These word limits may be taken as excluding the captions for diagrams or graphs, linking words between equations, and any appendices.)

Your report should have the following section headings, which are based on the stages of the modelling cycle:

Specify the purpose

Create the model

Do the mathematics

Interpret the results

Evaluate the model

Revise the model

Conclusions

Modelling problem – Yellow lines

When one is travelling by bus or car along a major road, one sometimes sees on the approach to a junction a succession of yellow lines painted across the carriageway. These warn the driver to slow down before reaching the junction. However, the lines are intended to do more than just warn: they are designed to encourage the driver to decelerate by creating the impression that the vehicle is going too fast otherwise.

In order to do this, the lines are positioned progressively closer together as they get nearer to the junction. Crossing the lines provides a very strong visual clue to a vehicle's speed (and also an auditory clue, as the lines are usually painted so as to produce a click as the vehicle passes over them). If a vehicle is approaching the junction at a constant speed, then the lines come past more and more quickly, giving the driver the feeling that the vehicle is accelerating. To compensate, the driver will tend to drive so that the lines come past at a constant rate. Thus drivers can be encouraged to slow down by careful spacing of the lines.

Create a model for suggesting both the number and the spacing of the lines in order to take the greatest advantage of this effect.

General advice

The mark scheme for this report is given on pages 5–6. The number of marks allocated to each section gives an indication of how much is expected there.

Usually, you should base your report on the model that you derived with your modelling group. Talk to your tutor if you wish to do otherwise.

Your report will be read (and marked) by your tutor, but you may find it helpful to imagine writing it for another MST210 student to read. Use this as a guide for how much mathematical detail to include.

Your report should explain to your tutor clearly and fully what results you obtained and how you obtained them. Remember that your tutor cannot award marks for work that he or she cannot comprehend or for steps that are missing. It is your responsibility to explain your work satisfactorily.

You do not need to aspire to exacting standards of literary style. Your tutor will not worry unduly about occasional spelling mistakes. On the other hand, you should be concerned with those aspects of writing that are closest to mathematics: being systematic; laying out your work in a logical fashion; making sure that each step of an argument or calculation follows from the previous one; avoiding any obscurity or ambiguity in the use of symbols and technical terms; using appropriate diagrams and tables. Moreover, no piece of writing is going to make much sense if it contains lots of meaningless or ambiguous sentences, so you should also ensure that what you write is clearly expressed. Take care to be consistent in your choice of words and symbols.

Your report does not have to be *drafted* in the order in which it will be finally presented and read. Most people find it quite difficult to write anything longer than a postcard by starting at the beginning and keeping steadily on until the end is reached. It is usually necessary to make at least one rough draft, and to work backwards and forwards through the draft as the ideas develop. If you proceed in this way, then you will find that the resulting account of your model is more comprehensive and better structured than if you just start at the beginning and carry on to the end.

Take every opportunity to include figures and illustrations in your report. You can give sketches of graphs, drawings of situations, diagrams of processes and relationships, and computer output. Explanations of complicated derivations are often easier to understand if they are accompanied by appropriate diagrams. Data, numerical results and lists of variables are usually best displayed in tables.

You may find it helpful during your later work on this report to find one or two (non-mathematical) friends who might be interested in the problem that you are going to solve. Persuade them to be your *audience* and ask them to read your report when first drafted. Your audience will be useful for discussion when you become stuck, and should help you to produce a report that is clear and easy to understand.

Before submission, you may like to check that you have:

- given a clear statement of the problem to be solved
- outlined the main features of your model, without using detailed mathematics
- given a numbered list of clearly stated assumptions
- removed any data items from the list of assumptions
- ensured that assumptions are referenced where they are used
- removed any assumptions that are not used in the model
- added any assumptions that have been used in the model but not explicitly stated
- added definitions for all variables and parameters
- written the report using good prose
- derived the first model as far as possible without including data values
- checked that the results are sensible
- checked that your model is reasonable in any special cases
- produced credible predictions from your model

Mark scheme

Marks are allocated to the group work as follows.

Group work

Modelling forum. 1 mark for any posting on the modelling forum. Up to 2 further marks for extensive engagement with the forum.

Modelling wiki. 1 mark for any edit of the modelling wiki. An additional mark is available for greater engagement with the wiki.

Marks are assigned to each section of your report. Under each section heading are instructions about what the section should contain. These instructions are not meant to be followed slavishly: they are just a guide to writing a good report.

Specify the purpose

Define the specific problem to be solved. Write a clear, succinct statement of the specific problem addressed in your report, in your own words (do not just repeat the given problem statement).

Create the model

Describe features investigated and outline mathematics used

Mention the features from your initial feature list that you are investigating. Give a qualitative description of the approach to be used in the first model, to explain why and how the first model will be formulated.

State assumptions **7**

Create a numbered list of clearly stated assumptions used in the model (take care not to miss assumptions or include assumptions that are never used). Do not attempt to justify assumptions here. Data values should not appear in assumptions, so, for example, 'the width of the road is 10 m' should be replaced by 'the width of the road is constant'.

Choose variables and parameters

Create a table of all symbols used in the model. For each symbol, state a clear definition and its associated units. It is not necessary to distinguish between variables and parameters.

Formulate mathematical relationships

Derive relationships between your variables and parameters. You should explain how the equations follow from your assumptions, which should be referenced.

Do the mathematics

Derive a first model. Solve your first model to find the variable of interest (as specified in the purpose of the model) in terms of other variables and parameters. Clearly state the mathematical model derived. It is not necessary to have one overall explicit equation; it is possible to have a series of equations, which may aid clarity, or an implicit equation (that will be solved numerically). Your solution at this stage should not include particular data values.

Draw graphs showing typical relationships. Sketch graphs to show the expected variation of variables predicted by your model. Use typical values for any parameters.

Check your model using dimensional analysis.

Interpret the results

Collect relevant data for parameter values. Usually, relevant data are available on the internet (or in the library), in which case the source should be referenced. If your data are from a simple experiment, then state the results here and describe the experiment in an appendix.

Describe the mathematical solution. Substitute data into your model to find solution(s). Clearly state in words this solution and how it relates to the purpose of the model. This should be written in a form that could be understood by a lay-person, by presenting it, for example, as a set of instructions, a graph or a table.

Find predictions to compare with reality. Look for any predictions of your model, or part of your model, or a corollary of your model that may be tested.

Evaluate the model

Collect data to compare with the model. Collect additional data to test your model. Do not use the data used to define parameter values. Usually, the additional data will be from the internet (or the library) and should be referenced. If your data are from a simple experiment, then state the results here and describe the experiment in an appendix.

Test your first model. Compare model predictions with your additional data. Some models may be impossible to test in this way, in which case you should explain why it is not possible to test your model. Marks are available for describing a test without actually being able to perform it.

Criticise your first model. Criticise your model based on the tests that you performed.

Review your assumptions. Consider each assumption in turn, and explain what would be the effect of changing it. Focus on those assumptions that would improve the fit to the evaluation data.

Revise the model

Decide whether to revise your first model. Decide whether a revision of your first model is justified. Explain why you made your decision, referring to the evaluation of the first model and your review of the assumptions. If your first model fits your data well, then consider if a simpler model might be better.

Describe your intended revision. Include a clear statement of any assumptions that are being revised and the new assumption(s) that will replace them. Note that a change of a parameter value does not constitute a revision of the model. Try to explain how the revision you suggest might affect any differences between the predictions of the first model and the data used for evaluation.

Conclusions

Summarise your modelling. Include the performance of your first model, any attempts to improve on it, and any comments on the modelling process. This short summary should not introduce any new considerations.