CHG1371

PROJECT DESCRIPTION

1. Problem

A carbon steel cylinder, initially at temperature T_0, is placed into a room with ambient temperature, T_∞. Over time the cylinder approaches the temperature of the room. A schematic of the cylinder is shown in Figure 1 below. This problem consists of a 2 dimensional heat diffusion problem (neglecting heat transfer in the angular direction).

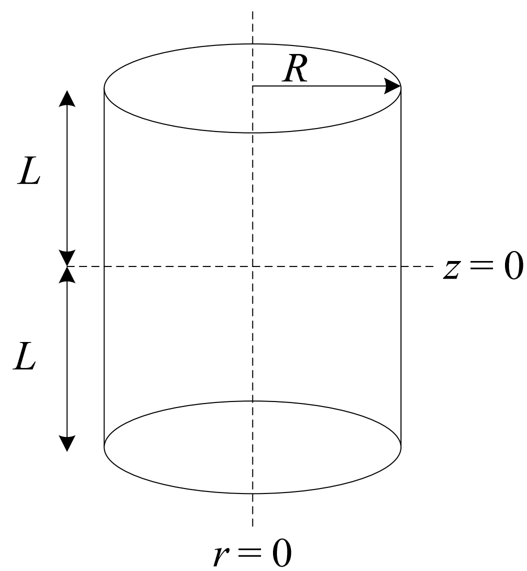

Figure 1: Schematic of the cylinder when $t < 0, T = T_0$, and when $t \geq 0, r = R, z = \pm L \quad T = T_\infty$.

The governing equation is

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) \qquad [1]$$

where α is the thermal diffusivity, $\alpha = k/(\rho c_P)$, which is a function of the thermal conductivity, k, the density, ρ, and the heat capacity, c_P, of the solid.

Equation 1 can be described by the dimensionless temperature, $\hat{T}$, such that

$$\hat{T} = \frac{T_\infty - T}{T_\infty - T_0} \qquad [2]$$

Equation 1 then becomes

$$\hat{T}(t,r,z) = \frac{\partial \hat{T}}{\partial t} = \alpha\left(\frac{\partial^2 \hat{T}}{\partial r^2} + \frac{1}{r}\frac{\partial \hat{T}}{\partial r} + \frac{\partial^2 \hat{T}}{\partial z^2}\right) \qquad [3]$$

Assuming that the variables z and r can be separated Equation 3 can be expressed as

$$\hat{T}(t,r,z) = \Gamma(t,r)\Psi(t,z) \qquad [4]$$

In Equation 4, $\Gamma(t,r)$ represents a function which is dependent only on time and radius and $\Psi(t,z)$ represents a function which is only dependent on time and height. Taking the derivative of Equation 4 and combining with Equation 3 gives

$$\Psi\frac{\partial \Gamma}{\delta t} + \Gamma\frac{\partial \Psi}{\delta t} = \alpha\Psi\left[\frac{1}{r}\frac{\partial \Gamma}{\partial r} + \frac{\partial^2 \Gamma}{\partial r^2}\right] + \alpha\Gamma\frac{\partial^2 \Psi}{\delta z^2} \qquad [5]$$

Equation 5 can be rearranged to give

$$0 = \Psi\left[\alpha\frac{1}{r}\frac{\partial \Gamma}{\partial r} + \alpha\frac{\partial^2 \Gamma}{\partial r^2} - \frac{\partial \Gamma}{\partial t}\right] + \Gamma\left[\alpha\frac{\partial^2 \Psi}{\partial z^2} - \frac{\partial \Psi}{\partial t}\right] \qquad [6]$$

Equation 6 can only be solved if

$$\frac{\partial \Gamma}{\partial t} = \alpha\left(\frac{1}{r}\frac{\partial \Gamma}{\partial r} + \frac{\partial^2 \Gamma}{\partial r^2}\right) \qquad [7]$$

and

$$\frac{\partial \Psi}{\partial t} = \alpha\frac{\partial^2 \Psi}{\partial z^2} \qquad [8]$$

Equation 7 can be converted into a dimensionless equation with Fourier number, $\tau_R = \alpha t / R^2$, and dimensionless radius, $\hat{r} = r/R$ giving

$$\frac{\partial \Gamma}{\partial \tau_R} = \frac{1}{\hat{r}}\frac{\partial \Gamma}{\partial \hat{r}} + \frac{\partial^2 \Gamma}{\partial \hat{r}^2} \qquad [9]$$

Equation 9 describes the temperature profile of a cylinder in the radial direction and has an analytical solution.

$$\Gamma(t,r)=\sum_{m=1}^{\infty}\left[\frac{2J_1(\lambda_m)}{\lambda_m\left[J_0^2(\lambda_m)+J_1^2(\lambda_m)\right]}\left[J_0(\lambda_m\hat{r})e^{-\lambda_m^2\tau_R}\right]\right] \qquad [10]$$

where λ_m is the m^{th} root of Equation 11 below, and $J_n(x)$ is the Bessel function of the first kind (also called the Bessel J function).

$$\lambda\frac{J_1(\lambda)}{J_0(\lambda)}=\frac{hR}{k} \qquad [11]$$

where h is the convective heat transfer coefficient of the surrounding fluid.

Equation 8 can also be converted into a dimensionless equation with Fourier number, $\tau_L=\alpha t/L^2$, and dimensionless radius, $\hat{z}=z/L$ giving:

$$\frac{\partial\Psi}{\partial\tau_L}=\frac{\partial^2\Psi}{\partial\hat{z}^2} \qquad [12]$$

Equation 12 describes the temperature profile of a slab in the z direction and also has an analytical solution.

$$\Psi(t,r)=\sum_{n=1}^{\infty}\left[\frac{2\sin(\lambda_n)}{\lambda_n+\sin(\lambda_n)\cos(\lambda_n)}\left[\cos(\lambda_n\hat{z})e^{-\lambda_n\tau_L}\right]\right] \qquad [13]$$

where λ_n is the n^{th} root of Equation 14 below:

$$\lambda\tan(\lambda)=\frac{hL}{k} \qquad [14]$$

Therefore, the final solution using Equations 4, 10 and 13 is

$$\hat{T}(t,r,z)=\sum_{m=1}^{\infty}\left[\frac{2J_1(\lambda_m)}{\lambda_m\left[J_0^2(\lambda_m)+J_1^2(\lambda_m)\right]}\left[J_0(\lambda_m\hat{r})e^{-\lambda_m^2\tau_R}\right]\right]\sum_{n=1}^{\infty}\left[\frac{2\sin(\lambda_n)}{\lambda_n+\sin(\lambda_n)\cos(\lambda_n)}\left[\cos(\lambda_n\hat{z})e^{-\lambda_n\tau_L}\right]\right] \qquad [15]$$

2. Task

The task of this project is to create a profile of the temperature of the centre (r = 0, z = 0) of the cylinder as a function of time. Also find the time when the centre of cylinder reaches the ambient temperature.

The solution **must be solved in Microsoft Excel** The solution should be flexible and robust enough that different parameters (e.g. different k, R, h, etc.) could be easily changed and a new solution would be automatically calculated. At least two different root finding methods should be attempted and the efficiency of the method should be discussed in the report. The numerical methods should also be able to be easily adapted to handle different nonlinear problems. For this problem the following parameters in Table 1 will be used.

Table 1: Parameters to test.

Variable	Value	Unit
R	0.3	m
$2L$	1.2	m
T_0	750	K
T_∞	300	K
c_P	420	J/kg·K
h	30	W/m^2·K
ρ	7850	kg/m^3
k	43	W/m·K