EE 453: Homework 4
Due in PDF on Canvas: 9 April 2021

Fundamental Problems (20 pts)

1. Consider the impulse response $h[n] = 0.25\delta[n-1] + 0.5\delta[n] + 0.25\delta[n+1]$
   (a) Determine the magnitude respnose $|H(e^{j\omega})|$
   (b) What is the phase response of this filter? How can you obtain a **linear-phase** filter from this $h[n]$?
   (c) Obtain a length three **linear-phase highpass** filter by suitably modifying the coefficients of the linear-phase version of $h[n]$.

2. Consider a **stable, causal** IIR transfer function with squared-magnitude response given by

$$|H(e^{j\omega})|^2 = \frac{9(1.09 + 0.6\cos\omega)(1.25 - \cos\omega))}{(1.36 + 1.2\cos\omega)(1.16 + 0.8\cos\omega)}$$

   $|H(e^{j\omega})|^2 = H(z)H(z^{-1})|_{z=e^{j\omega}}$ **HINT:** $\cos\omega \mapsto \frac{1}{2}(z + z^{-1})$
   (a) Determine a **stable** transfer function $H(z)$ such that $H(z)H(z^{-1})|_{z=e^{j\omega}}$ satisfies the above squared-magnitude response
   (b) How many **stable, distinct** transfer functions $H_i(z)$ are there such that:

$$H_1(z)H_1(z^{-1})|_{z=e^{j\omega}} = H_2(z)H_2(z^{-1})|_{z=e^{j\omega}} = ... = H_n(z)H_n(z^{-1})|_{z=e^{j\omega}}$$

   (c) Among the different transfer functions $H_i(z)$, identify the minimum-phase, mixed-phase, and maximum-phase systems.
   (d) Plot the different pole-zero diagrams for each different transfer function, again identifying minimum/maximum/mixed phase
   (e) Calculate the all-pass filter which transforms the minimum-phase filter into the maximum-phase filter

3. Consider the IIR filter whose transfer function is given by

$$H(z) = \frac{(0.2z^{-1} + 0.4z^{-2})(3 - 2.2z^{-1})}{(2 - 3z^{-1} + 4z^{-2})(1 - 0.7z^{-1})}$$

   (a) Draw the (i) Direct Form I, (ii) Direct Form II, (iii) Direct Form II Transpose representations for $H(z)$
   (b) Draw two different cascade realizations of $H(z)$ using second-order sections
   (c) Implement the following Parallel Forms for this system function:
      - Parallel Form I: $H(z) = 0.3143 - \frac{0.0268}{1-0.7z^{-1}} + \frac{-0.2875+0.7501z^{-1}}{1-1.5z^{-1}+2z^{-2}}$
      - Parallel Form II: $H(z) = \frac{0.0188z^{-1}}{1-0.7z^{-1}} + \frac{0.3188z^{-1}+0.5750z^{-2}}{1-1.5z^{-1}+2z^{-2}}$

4. For each of the following systems, the goal is to identify the associated minimum phase systems, so that $|H(e^{j\omega})| = |H_{min}(e^{j\omega})|$.
   (a) $H(z) = \frac{1-2z^{-1}}{1+\frac{1}{3}z^{-1}}$
   (b) $H(z) = \frac{(1+3z^{-1})(1-\frac{1}{2}z^{-1})}{z^{-1}(1+\frac{1}{3}z^{-1})}$
   (c) $H(z) = \frac{(1-3z^{-1})(1-\frac{1}{4}z^{-1})}{(1-\frac{3}{4}z^{-1})(1-\frac{4}{3}z^{-1})}$

Advanced Problems (40 pts)

5. Consider the IIR transfer function

$$H(z) = \frac{1 + \frac{3}{16}z^{-1}}{1 + \frac{3}{8}z^{-1} - \frac{5}{32}z^{-1}}$$

   Suppose the fractional coefficients are quantized by rounding up to three fractional bits, so that $[\frac{1}{16} \to \frac{3}{16}) \mapsto \frac{1}{8}$, $[\frac{3}{16} \to \frac{5}{16}) \mapsto \frac{2}{8}$, etc.

   (a) Determine the Direct Form II system function for $H(z)$ (Taking into account the quantization, **You do not need to draw it**)

   (b) Determine the Parallel system function for $H(z)$ (Taking into account the quantization, **You do not need to draw it**)

   (c) Determine the Cascade system function for $H(z)$ (Taking into account the quantization, **You do not need to draw it**)

   (d) Which representation in (a) - (c) had the least error due to quantization?

6. Suppose that you want to deisgn a discrete-time lowpass Butterworth filter to meet the following specifications: $F_p = 500$ Hz, $F_{stop} = 1000$ Hz, with a minimum stopband attenuation of 50 dB.

   (a) Using **Impulse Invariance**, determine the minimum filter order for the following sampling frequencies: 10 kHz, 5 kHz, 2.5 kHz

   (b) Using **Bilinear Transformation**, determine the minimum filter order for the following sampling frequencies: 10 kHz, 5 kHz, 2.5 kHz

7. (a) Prove that **impulse invariance** maps the $j\Omega$ axis in the s-plane to the unit circle in the z-plane

   (b) Prove that **impulse invariance** preserves stability

   (c) Prove that the **bilinear transformation** maps the $j\Omega$ axis in the s-plane to the unit circle in the z-plane

   (d) Prove that the **bilinear transformation** preserves stability